JOURNAL OF THE AMERICAN

ROCKET

SOCIETY

A journal devoted to rocket technology and the jet propulsion sciences

STACKS

VOLUME 23 NOVEMBER-DECEMBER 1953 NUMBER 6

Kinetics of the Combustion Wave . . . . . *Raymond Friedman* 349

Determination of Thrust Alignment in Rocket Engines . . . . *W. L. Rogers* 355

Minimal Rocket Trajectories . . . . . *D. F. Lawden* 360

The Stability of Ethylene Oxide . . . . . *E. Milton Wilson* 368

Technical Notes 370
Zwicky, on Scintillations of Stars
Ehricke, on Take-Off From Satellite Orbits
Stehling and Kovitz, on Rocket Propellants

Jet Propulsion News 375

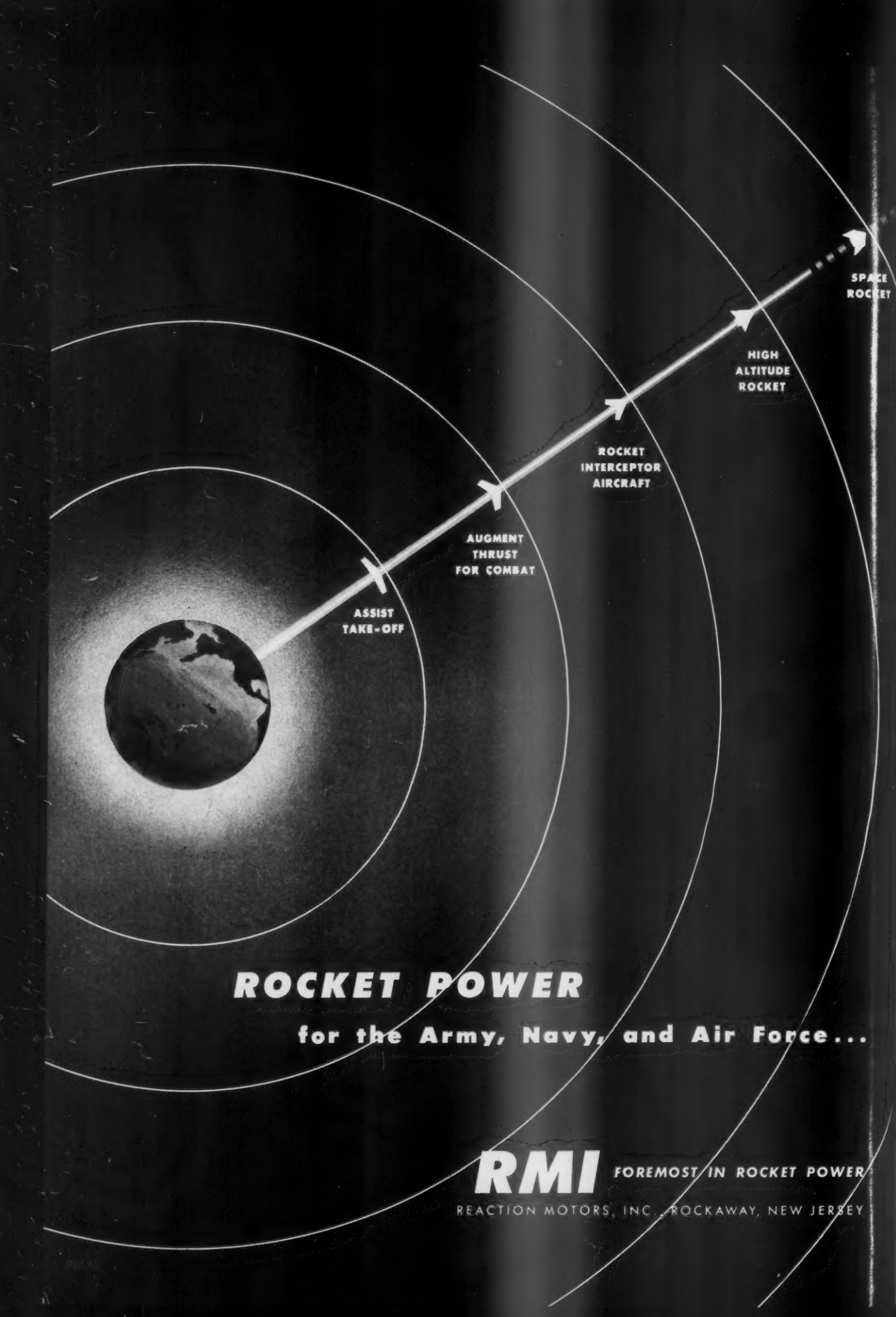
American Rocket Society News 383

Book Reviews 387

Technical Literature Digest 390

Convention Issue

ARS 8th ANNUAL CONVENTION! NEW YORK, DEC. 2–4 PROGRAM ON PAGE 383



ROCKET POWER

for the Army, Navy, and Air Force...

RMI FOREMOST IN ROCKET POWER

REACTION MOTORS, INC., ROCKAWAY, NEW JERSEY

JOURNAL OF THE AMERICAN
ROCKET
SOCIETY

PUBLICATION OFFICE:
20th and Northampton Sts., Easton, Pa.

EXECUTIVE OFFICES:
Engineering Societies Building
29 West 39th Street, New York 18, N. Y.

EDITOR-IN-CHIEF
MARTIN SUMMERFIELD
Princeton University

ASSOCIATE EDITORS
A. J. ZAEHRINGER—*Jet Propulsion News*
ACTING EDITOR
Thiokol Corporation

H. K. WILGUS—*ARS News*
New York, N. Y.

H. S. SEIFERT—*Book Reviews*
California Institute of Technology

ASSISTANT EDITOR
IRVIN GLASSMAN
Princeton University

MANAGING EDITOR
H. K. WILGUS
New York, N. Y.

EDITORIAL BOARD

D. ALTMAN
California Institute of Technology

L. CROCCO
Princeton University

P. DUWEZ
California Institute of Technology

R. D. GECKLER
Aerojet Engineering Corporation

C. A. GONGWER
Aerojet Engineering Corporation

C. A. MEYER
Westinghouse Electric Corporation

P. F. WINTERNITZ
Reaction Motors, Inc.

K. WOHL
University of Delaware

M. J. ZUCROW
Purdue University

ADVISORS ON PUBLICATION POLICY

L. G. DUNN
Director, Jet Propulsion Laboratory
California Institute of Technology

T. C. FETHERSTON
General Publicity Department
Union Carbide and Carbon Corporation

R. E. GIBSON
Director, Applied Physics Laboratory
Johns Hopkins University

H. F. GUGGENHEIM
President, The Daniel and Florence
Guggenheim Foundation

LOVELL LAWRENCE, JR.
Engineering Consultant

D. L. PUTT
Major General, U.S. Air Force
Commander, Air Research and
Development Command

T. VON KÁRMÁN
Chairman, Scientific Advisory Board
U.S. Air Force

W. E. ZISCH
General Manager
Aerojet Engineering Corporation

OFFICERS

President	F. C. Durant, III
Vice-President	Andrew G. Haley
Executive Secretary	James J. Harford
Secretary	A. C. Slade
Treasurer	Robert W. Lawrence
General Counsel	Andrew G. Haley

BOARD OF DIRECTORS

Three-year term of office expiring on dates indicated

Kurt Berman, 1955	R. W. Porter, 1953
Noah S. Davis, 1955	C. C. Ross, 1953
Roy Healy, 1955	Martin Summerfield, 1954
G. Edward Pendray, 1954	Raymond Young, 1953
M. J. Zucrow, 1954	

ARS Journal to be re-named JET PROPULSION

Beginning with the January-February 1954 issue, the full name of the ARS Journal will become "JET PROPULSION, the Journal of the American Rocket Society." This change is being made after much thought and research by the officers of the ARS.

First, let me reassure our space flight proponents that the ARS has not changed its aims or interest in space flight. However, it is now widely recognized that many of the basic scientific problems relating to space flight are being studied in all realms of jet propulsion. The solution to many problems in rocketry are being obtained in the turbojet and ramjet powered aircraft and guided missile programs.

For example, the basic data that are needed to solve aerodynamic and structural problems of a supersonic guided missile or the combustion problems of supersonic ramjets are essentially the same data required before we can design and develop an artificial satellite. We can never design a vehicle to return to the earth's surface from the conditions of speed and altitude required for satellite flight until we have first solved those problems more immediately besetting the aviation and guided missile industry.

I believe that a closer realization of today's high-speed, high-altitude problems will help my brother space flight proponents to understand why we should be patient. At the same time, I believe that many serious researchers in the aviation and guided missile industries are now fully aware of how their current problems are related directly to the achievement of space flight.

There is today no professional scientific journal in the United States which considers solely the problems of high-speed, high-altitude flight (and its logical conclusion—space flight). The ARS plans to provide a journal which focuses attention on these manifold problems.

Therefore, the Journal of the American Rocket Society, starting in 1954, will be called JET PROPULSION. No change in editorial policy is contemplated at this time. The same format as the present issue, except for the replacement of the word "Rocket" by "Jet Propulsion," will be employed in future issues.

As scientific data slowly provide the basis for technology, we shall see, in logical succession, realization of the successive achievements required before man ventures into space.

F. C. Durant, III, President, American Rocket Society

Advertising Representatives

JAMES C. GALLOWAY
816 W. 5th St., Los Angeles, Calif.
Telephone: Mutual 8335

THORPE COVINGTON
7530 N. Sheridan Road, Chicago 26, Ill.
Telephone: Rogers Park 1-1892

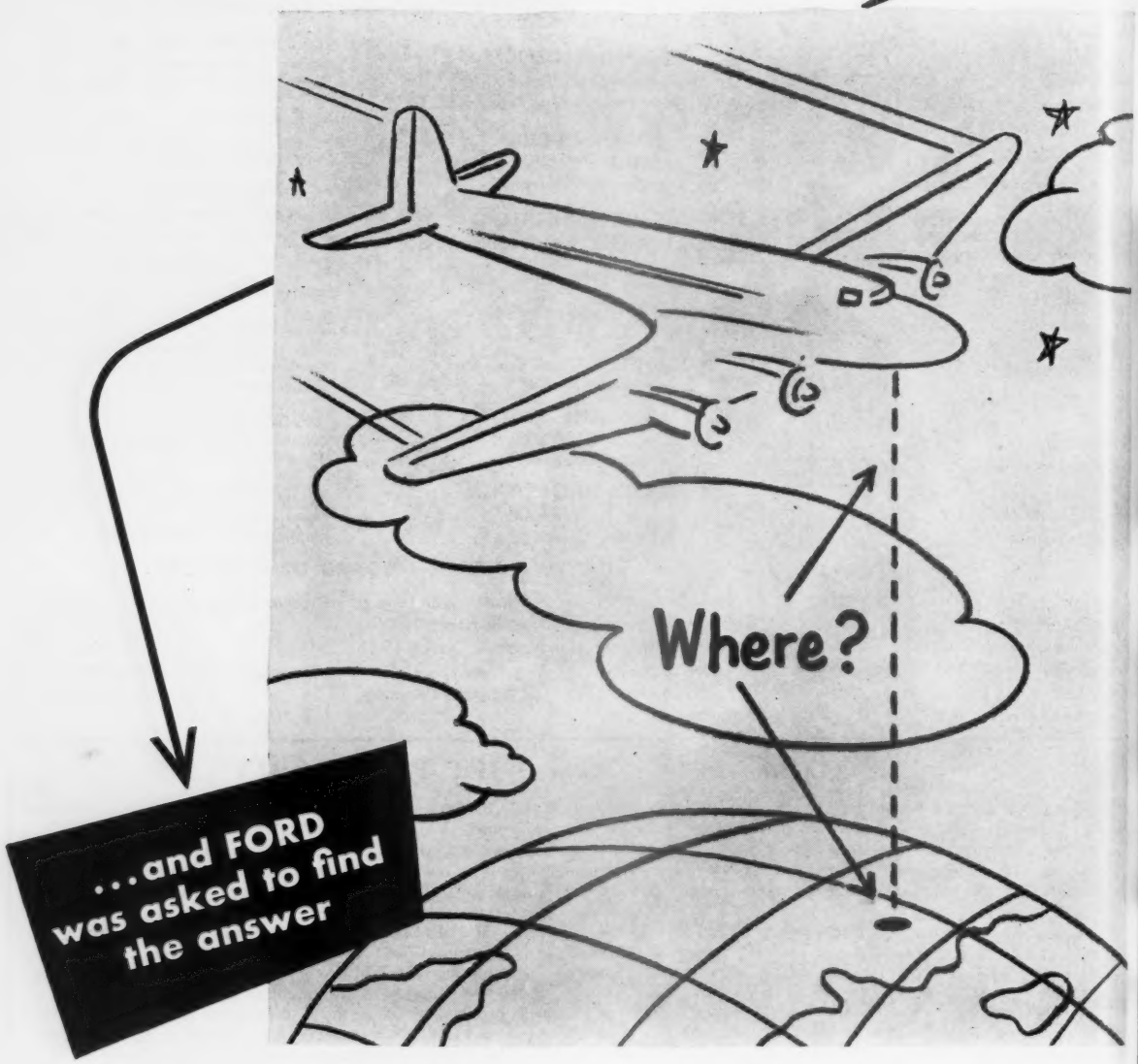
EMERY-HARFORD
155 East 42 St., New York, N. Y.
Telephone: Mu 4-7232

Published Bimonthly by the American Rocket Society, Inc.

Journal of the American Rocket Society, published bimonthly by the American Rocket Society at 20th and Northampton Streets, Easton, Pa., U.S.A. The Editorial Office is located at the Engineering Societies Building, 29 West 39th Street, New York 18, N. Y. Price $1.25 per copy. $7.50 per year. Entered as second-class matter at the Post Office at Easton, Pa., under the Act of March 3, 1879.

Copyright, 1953, by the American Rocket Society, Inc.

TO INDICATE aircraft position with no ground-to-air communication



Combat mission . . . or freight flight . . . now we are working to help the pilot locate his position without a radio beacon – merely by equipment right in the cockpit of his plane! Thanks to a Ford Instrument Company design, development and manufacture . . . another step is being taken toward greater flying safety.

This is typical of the problems that Ford has been given by the Armed Forces since 1915. For from the vast engineering and production facilities of the Ford Instrument Company, come the mechanical, hydraulic, electro-mechanical, magnetic and electronic instruments that bring us our "tomorrows" today. Control problems of both Industry and the Military are Ford specialties.

You can see why a job with Ford Instrument offers young engineers a challenge. If you can qualify, there may be a spot for you in automatic control development at Ford. Write for brochure about products or job opportunities. State your preference.

FORD

FORD INSTRUMENT COMPANY

DIVISION OF THE SPERRY CORPORATION

31-10 Thomson Avenue, Long Island City 1, N. Y.

LANDS WITH A 12-TON LOAD ...ON AN 855 FOOT STRIP!

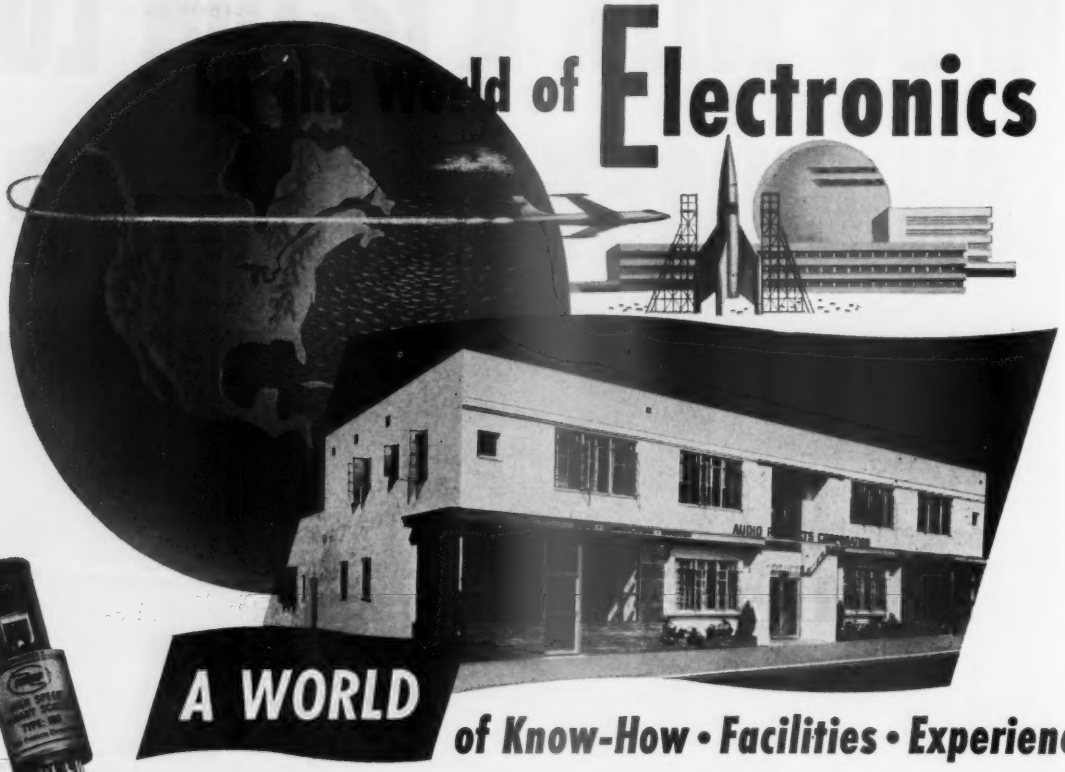
No cargo airplane ever built can get into and out of the tight spots with a big bulk load, like the combat-proven Fairchild "Flying Boxcar." The C-119 lands and brakes to a stop on an 855 foot runway with a 12-ton load! Designed in every detail as a general military bulk cargo transport, the C-119 is equipped with reversible propellers and a powerful main and nose wheel braking system that enable it to land on the aircraft equivalent of the proverbial "dime." Here is the medium transport cargo airplane that so exactly meets the combat-transport needs of the U. S. Armed Forces!

ENGINE AND AIRPLANE CORPORATION
FAIRCHILD
Aircraft Division
HAGERSTOWN, MARYLAND

Guided Missiles Division, Wyandanch, L. I., N. Y.
Engine Division, Farmingdale, L. I., N. Y.

RADAR • TELEMETERING • COMMUNICATIONS • NAVIGATIONAL AIDS • ELECTRONIC MINIATURIZATION

ld of Electronics



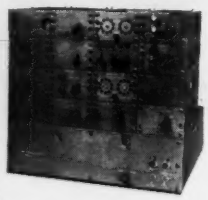
A WORLD of Know-How • Facilities • Experience

MINIATURIZED PACKAGED CIRCUITS

Plug-in circuitry in a compact unit. Custom-engineered to individual specifications based on circuit requirements or performance needs.

THE MODULAR SYSTEM

A basic electronic tool of great flexibility to aid in the design and use of pulse methods for information transmission, storage and computation.



Do you have an application in the fields of radar, telemetry, communications, navigational aids or electronic miniaturization? If so, it will pay you to consult AUDIO PRODUCTS CORPORATION.

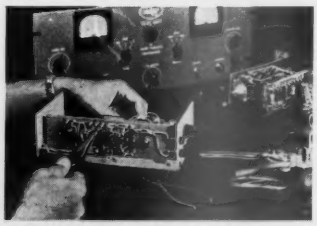
Here you'll find 35,000 square feet of modern production facilities and a staff of engineers accustomed to working in the more advanced phases of electronic development. From basic design to prototype, to final engineering tests—their knowledge, experience and facilities are applied to deliver a finished product to meet the highest standards of performance and reliability. A host of successful electronic units for military agencies and commercial organizations the world over is your proof. That is why we say—

For the Best in Electronic Design, Development and Manufacture, Consult

AUDIO PRODUCTS CORPORATION

2265 WESTWOOD BOULEVARD, LOS ANGELES 64, CALIFORNIA

The emphasis is on telemetry, radar relays, modulars and packaged circuitry



Highly skilled Audio Products technicians work with advanced production test facilities

Ever-expanding plant facilities provide more than 35,000 square feet of production space

NEW! A one-man Stretch-Wrap Former with instant work-length adjustment!

Designed to produce parts in an unusually wide range of work lengths, this new Hufford model 102 offers many work-saving features

ONE-MAN OPERATION—All loading and forming functions are easily controlled by a single operator.

INSTANT WORK-LENGTH ADJUSTMENT—Design has been reduced to its simplest form. Note work-length capacity is varied merely by extending or retracting cylinder rods to desired spacing between jaws.

WIDE RANGE OF LENGTHS—Parts as short as 12" or as long as 72" may be accommodated.

DOUBLING THE WORK-LENGTH CAPACITY is offered as a possibility by operating two machines side by side, disconnecting inside arms of each. Tension cylinder capacity remains the same.

SIMULTANEOUS ARM MOVEMENT—Arms may be automatically stopped at any portion of their travel by electric controls.

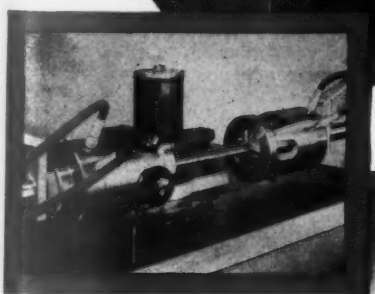
MANUAL TENSION CONTROL—Tension pressure is applied by manually operated levers.

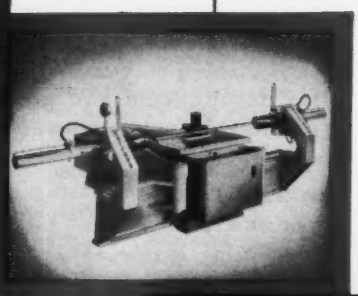
The new model 102 is Hufford's latest development towards modernization of existing equipment. It effectively relieves larger machines of the smaller jobs at considerable savings in original equipment cost. Quotations gladly furnished.

HUFFORD

MODEL 102

CAPACITY 5 TONS





▲ The model 102 easily and quickly forms parts over an unusual range of lengths. Since both arms pivot on the same pin small radii and short parts are easily and economically produced.

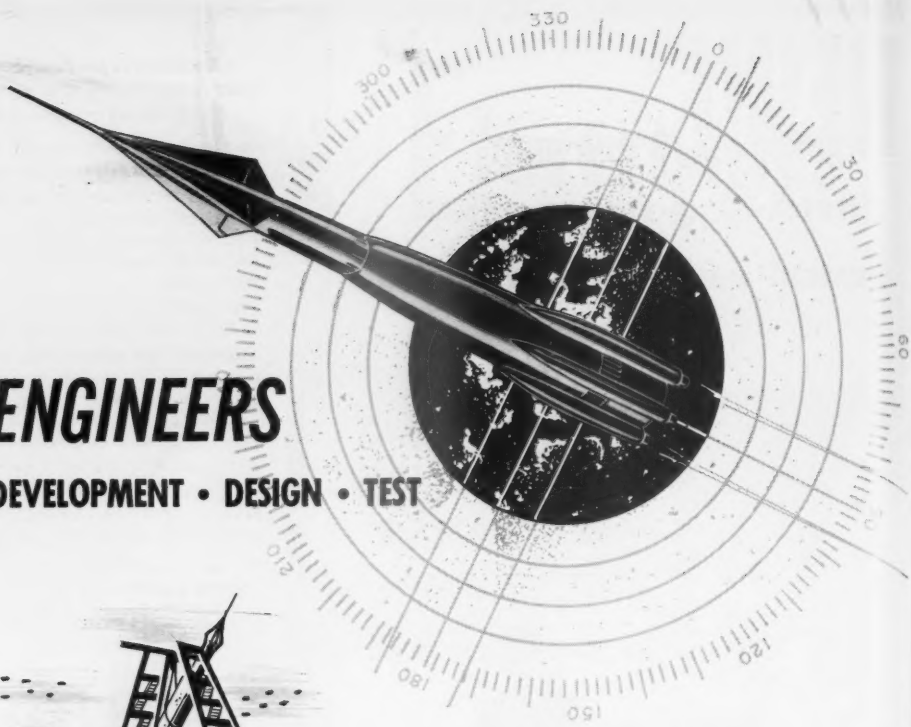
With jaws fully retracted, finished parts up to 72" long may be formed. ▶

BRIEF SPECIFICATIONS — MODEL 102

Tension Capacity	10,308 lbs.
Max. Die Height	10"
Min. Die Length for 180° Bend at Any Tonnage	7"
Max. Die Length for 180° Bend at Full Tonnage	45"
Tension cylinder stroke, each	30"
Maximum part length (tension cylinders fully retracted)	72"
Minimum part length	12"

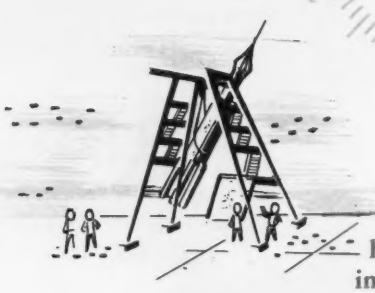
HUFFORD Machine Works Inc.
EL SEGUNDO, CALIFORNIA

Manufacturers of



ROCKET ENGINEERS

RESEARCH • DEVELOPMENT • DESIGN • TEST



A long range program of research and development in guided missiles has created unlimited opportunities in all phases of rocket engineering.

Engineers with advanced degrees are needed for positions in Combustion Research and Physical Chemistry.

Engineers with or without advanced degrees are needed as:

RESEARCH ENGINEERS . . . for studies in heat transfer and Thermodynamics

DESIGN ENGINEERS . . . for design phases of liquid rocket power plants, thrust chambers, gas turbine pumps

FIELD ENGINEERS . . . for coordination of activities at field test sites

TEST ENGINEERS . . . for development and production testing of liquid rocket power plants and their components

COMPLETE ROCKET TESTING FACILITIES

Openings also for Design Draftsmen and Technicians

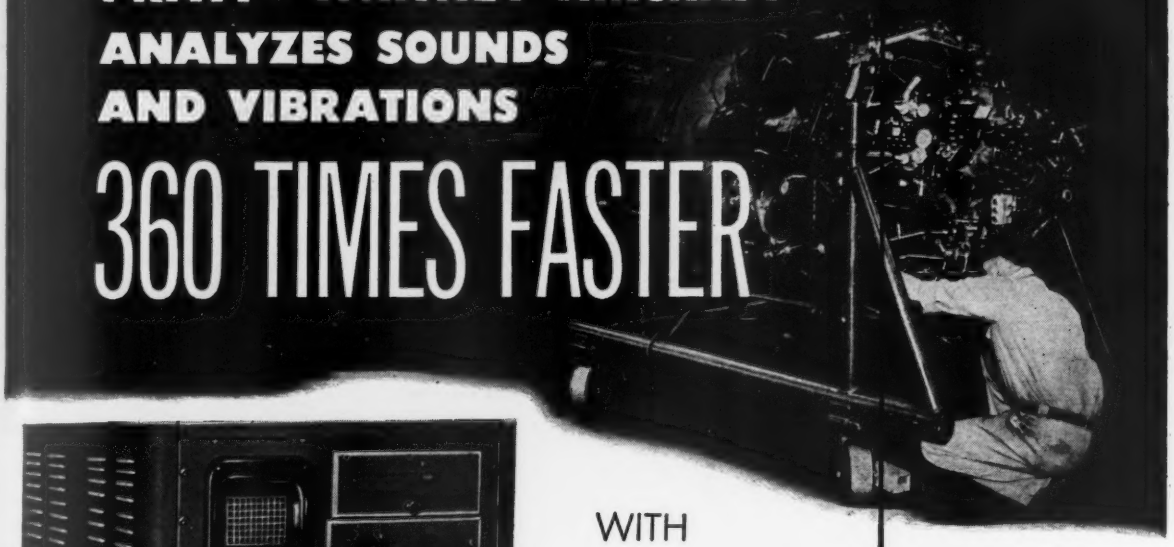
Send complete resume to: Manager, Engineering Personnel

BELL *Aircraft* CORPORATION

P O BOX 1, BUFFALO 5, NEW YORK

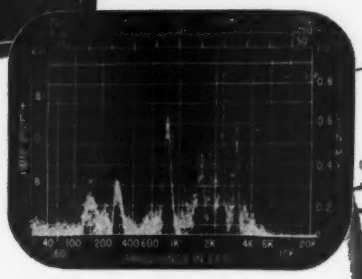
PRATT & WHITNEY AIRCRAFT ANALYZES SOUNDS AND VIBRATIONS

360 TIMES FASTER



WITH

PANORAMIC SONIC ANALYZER LP-1



Engaged in operations which demand maximum efficiency, Pratt & Whitney Aircraft has chosen the Panoramic Sonic Analyzer LP-1 to speed up and simplify sound and vibration analysis. The results of this careful selection are outlined in the Pratt & Whitney Aircraft report. The facts presented there speak for themselves.

In only one second, the LP-1 automatically visualizes on a Cathode Ray Tube the frequency and magnitude of sound and vibration components between 40 and 20,000 cps. The LP-1 is therefore an invaluable tool for analyzing random as well as steady phenomena. Special control features allow selection and magnification of narrow bands for detailed examination. Panoramic presentations are simple. They are readily interpreted by technical and non-technical personnel.

Whatever your problem . . . sound, vibration, or general spectrum analysis in the laboratory or on the production line, join the many top-flight manufacturing, research, and development activities which have increased their efficiency through the use of instruments made by Panoramic, the leader.

Report from

PRATT & WHITNEY AIRCRAFT

The Panoramic Sonic Analyzer Model LP-1 is being used by Pratt & Whitney Aircraft for analyzing spectra of pressure, vibration and noise measurements. It is found that records can be obtained in one minute that would take six hours by previously used methods. It is possible, therefore, to evaluate developmental changes more rapidly and to make more thorough analyses of available data.

The instrument is usually used with a Polaroid Land Camera for analyzing data recorded on tape recorders. For more immediate results visual analysis is used while the engine is operating and the more interesting points are then photographed.

- Operate with microphones, vibration pickups and other transducers. • Presents data graphically permitting rapid, direct readings of frequency and voltage. • Data can be easily photographed or recorded. • Linear and log voltage calibrations. • Simple operation.

the pioneer is the leader

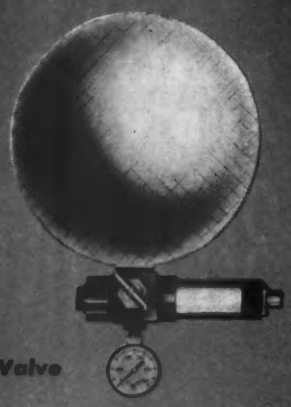
PANORAMIC RADIO PRODUCTS, INC.

WRITE FOR COMPLETE DETAILS, PRICES AND DELIVERY TODAY.

14 SOUTH SECOND AVE., MOUNT VERNON, N. Y. Phone: MOunt Vernon 4-3970

NEW MAROTTA VALVE

Provides Complete Control for Airborne Gas Pressure Bottles



MV-57 Solenoid Valve as installed on sphere bottle

MV-36 Solenoid Valve

MV-48A Magnetic Valve

PV-3 Propellant Shut-off Valve

Here in one compact light weight assembly is a complete control for gas pressure bottles . . . a vital contribution towards fast, trouble-free starts for jet engines. The MV-57 is an internally piloted, high capacity, 5 in 1 control valve, featuring on-off, pressure relief, drain, gauge and check operations in one safe, dependable unit that offers instant reaction at working pressures to 3500 psi. The weight saving factor in the MV-57 is significant. Also important are the reduction in procurement costs and the simplification of wiring and piping.

SPECIFICATIONS

Air Pressure	3500 psi
Operating Temperature .	−65° to +165°
Weight	1.8 lbs.

Pressure relief blows off at 3500 psi and resets at 3100 psi.
Pressure relief valve is provided with manual water drain

Marotta produces a complete line of highly specialized control valves. For further details on MV-57 or other Marotta valves write:

Marotta Valve Corp.

Boonton, New Jersey

World War II Veteran

holds its own in the new jet age

the Douglas B-26 Invader

During World War II, over both Europe and the Pacific, the Douglas B-26 Invader spearheaded major allied advances. It was still the standard USAF light bomber when war broke out in Korea.

Laminar-flow, high-speed wing design gives Invader the speed and maneuverability of a piston-engine fighter. With 3-man crew and 6,000-lb. bomb load, combat radius is over 900 miles. Firepower is crushing... sixteen .50 caliber machine guns, fourteen of them in the nose and wings. In Korea, despite the advent of fast new jets, Invader's agility and heavy armament have made it a mainstay in advanced, low-level ground support of our troops.

Performance of the B-26 Invader is another example of Douglas leadership. *Faster and farther with a greater payload* is a basic rule of Douglas design.

Enlist to fly in the U. S. Air Force

Depend on *DOUGLAS* **First in Aviation**

THE *STRATOS* DIVISION OF THE FAIRCHILD ENGINE AND AIRPLANE CORP. HAS 15 ENGINEERING POSITIONS OPEN

Stratos has interesting and challenging openings for qualified engineers with experience applicable to pneumatic accessories and high speed rotating equipment for industrial and aeronautical applications.

POSITIONS OPEN INCLUDE:

PROJECT ENGINEERS
(Seniors & Juniors)
CONTROLS ENGINEERS
RESEARCH ENGINEERS
DESIGN ENGINEERS

Interviews during the Rocket Society meetings can be arranged by calling Mr. R. T. Bartlett at Circle 7-7619 or Bay Shore 7-6500.

Excellent research and development facilities in a modern Air-conditioned plant. Good housing available. Ideal suburban living on Long Island, convenient to N. Y. C. Superior facilities for boating, golf, fishing, fine beaches, other recreational features. 5-day work week—extra compensation for extended work day.

SEND RÉSUMÉS TO

STRATOS

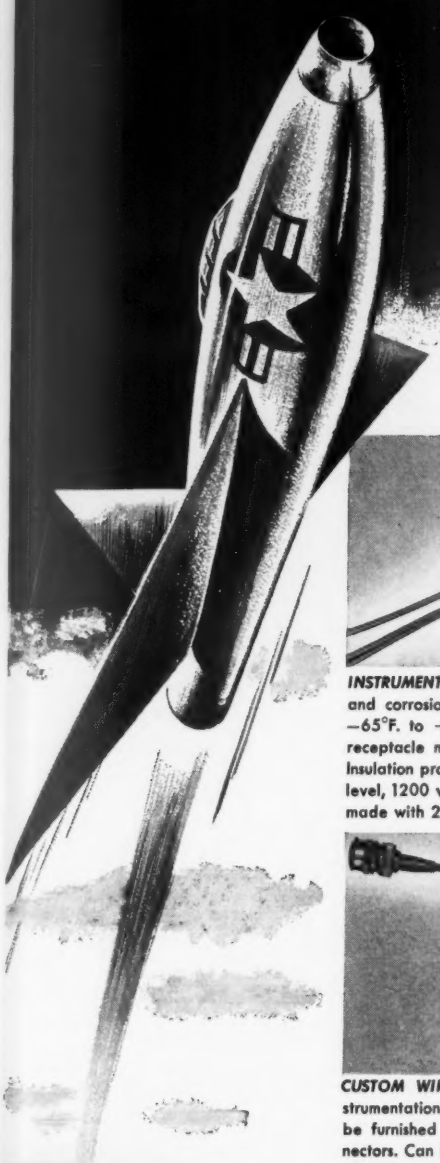
A DIVISION OF FAIRCHILD ENGINE & AIRPLANE CORP.

Main Office: Bay Shore, L. I., N. Y. • West Coast Office: 1355 Westwood Blvd., Los Angeles, Calif.

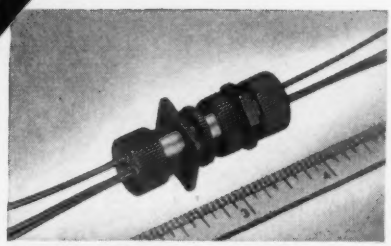
STRATOS IS DESIGNING AND PRODUCING SUCH EQUIPMENT AS:

REFRIGERATION TURBINE FOR FIGHTERS | **REFRIGERATION PACKAGE FOR JET BOMBERS** | **AIR-TURBINE DRIVE FOR ACCESSORIES** | **BOOTSTRAP COOLING UNIT FOR TRANSPORT AIRCRAFT** | **CABIN SUPERCHARGER FOR TRANSPORT AIRCRAFT**

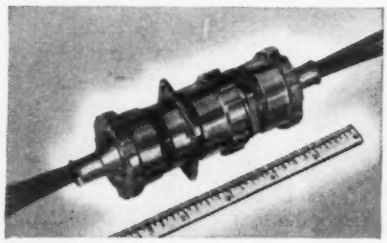
Extreme Temperature and Altitude Problems Solved by New Modifiable Titeflex Connector



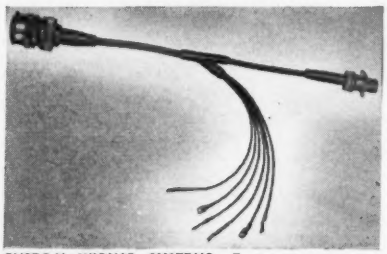
We have the experience to solve most complex connector problems involving extreme altitudes, temperatures and pressures with space and weight limitations. And within standard design requirements, we can develop special-duty connectors as a part of complete wiring systems.



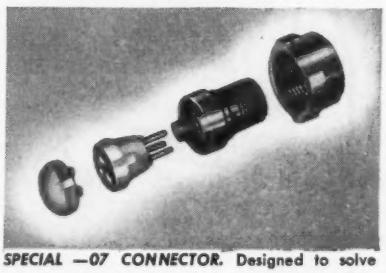
INSTRUMENTATION Connector —07. For moisture and corrosion resistance, temperature ranges of −65°F. to +400°F. Made of Teflon, plug and receptacle mated weigh only ⅔ oz. Length 2". Insulation properties will permit 3500 volts at sea level, 1200 volts at 50,000 feet altitude. Can be made with 2 or 3 pins, current 7 amperes.



MOISTURE-PROOF and resistant to synthetic lubricants. For extreme temperature changes in ranges of −65°F. to +400°F., high altitudes up to 65,000 feet. Resists salt spray, corrosion, vibration. This Titeflex Connector is radio shielded, has positive retention of pins and sockets. 5" in length. Mates with connectors that conform to MIL-C-5015.



CUSTOM WIRING SYSTEMS—For accessory, instrumentation and radio shielded applications. Can be furnished with Titeflex or Standard AN Connectors. Can be sheathed with one or more layers of various metal braids, fiber glass or nylon, and jacketed with silicone or various other compounds. Titeflex will be glad to design, develop and produce complete wiring systems to your specifications.



SPECIAL —07 CONNECTOR. Designed to solve your connector problems in instrumentation with a real saving in space and weight since this connector has no protuberance beyond the flange. Can be designed as an integral part of your wiring or instrument components. Available in 1, 2 or 3 pin arrangements—current 7 amperes, size 1" in length. Receptacle and plug weigh only 11 grams.

All TITEFLEX Connectors can be furnished with thermocouple pins and sockets.

WRITE TODAY for specific information—or send us your specifications. Whatever your requirements, we can usually provide the right answer. Our Engineering Staff will be glad to discuss your problem without obligation.

LET OUR FAMILY OF PRODUCTS HELP YOURS

Titeflex

TITEFLEX, INC.
578 Frelinghuysen Avenue, Newark 5, N. J.

MAIL COUPON TODAY

Please send me your catalog on the Titeflex Connector.

Have your representative call ☐.

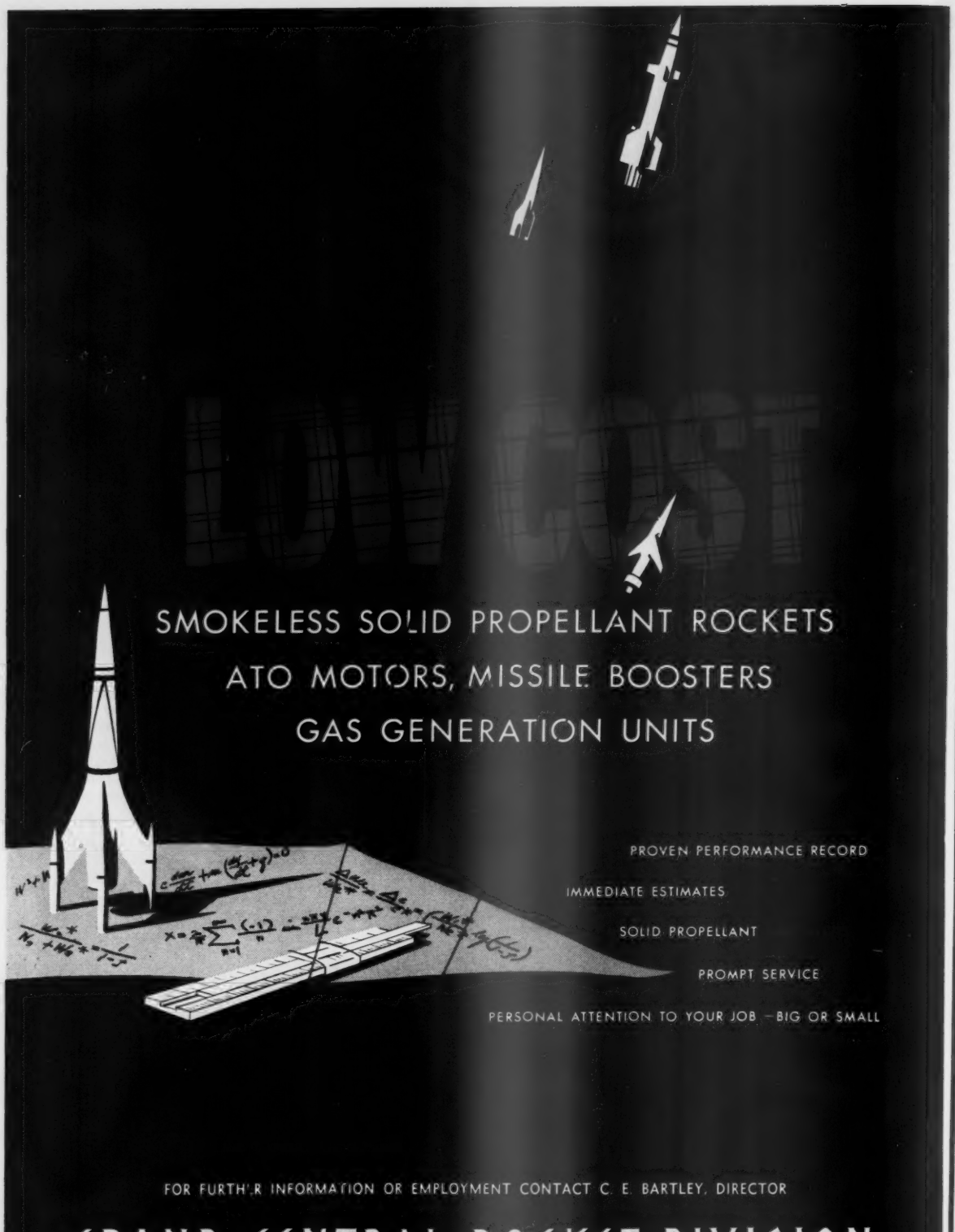
NAME_____

TITLE_____

COMPANY_____

ADDRESS_____

CITY_____ ZONE_____ STATE_____


SMOKELESS SOLID PROPELLANT ROCKETS
ATO MOTORS, MISSILE BOOSTERS
GAS GENERATION UNITS
PROVEN PERFORMANCE RECORD
IMMEDIATE ESTIMATES
SOLID PROPELLANT
PROMPT SERVICE
PERSONAL ATTENTION TO YOUR JOB — BIG OR SMALL
FOR FURTHER INFORMATION OR EMPLOYMENT CONTACT C. E. BARTLEY, DIRECTOR
GRAND CENTRAL ROCKET DIVISION
GRAND CENTRAL AIRCRAFT CO.
GLENDALE 1, CALIFORNIA

Name of ARS Journal will be **JET PROPULSION**

Starting with the January-February 1954 issue, the full name of the ARS JOURNAL will become "JET PROPULSION, the Journal of the American Rocket Society." See page 337 for further details.

Kinetics of the Combustion Wave

RAYMOND FRIEDMAN[1]

Westinghouse Research Laboratories, East Pittsburgh, Pa.

Theoretical concepts of one-dimensional steady-state combustion waves are discussed in a form which should be intelligible to those other than specialists in the field. Although a brief review of the fundamentals of detonation is included, the main emphasis is on deflagration. The aim of the presentation is to emphasize current developments rather than to give a historical review.

Nomenclature

k = frequency factor in reaction rate law
p = pressure
t = time
v = gas velocity
x = distance co-ordinate in flow direction
y_i = mole fraction of species i
z_i = fraction of mass rate of flow due to species i (Equation [4.6])
C_p = heat capacity per unit mass at constant pressure
D = diffusion coefficient
E = activation energy in reaction rate law
H = enthalpy per unit mass
L = reaction zone thickness; see Fig. 2
Q = mean heat release rate per unit volume
R = universal gas constant
T = absolute temperature
U = internal energy per unit mass
V_i = diffusion velocity of species i
X = quenching distance
ρ = density
λ = thermal conductivity
ϵ = E/RT
κ = $\lambda/C_p\rho$ (thermal diffusivity)
α = κ/D (Lewis number)
β = $\kappa_u k/v_u^2$

Subscripts

b = completely burned gas
u = unburned gas
0 (see Fig. 2)

1 Introduction

CONSIDER either a pure gas or a uniform gas mixture, which is capable of exothermic chemical reaction and which is flowing in the $+x$ direction, initially at velocity v_u, absolute static temperature T_u, and static pressure p_u. Now, let us assume that this gas passes through a transition region, on the other side of which the velocity, temperature, and pressure have new values v_b, T_b, and p_b. (The subscripts u and b refer to unburned and burned gas.) Let this be a steady-state process; i.e., if we fix our co-ordinate system relative to the transition region, then $v(x)$, $T(x)$, and $p(x)$ are independent of time. Finally, we assume that the burned gas is in a condition of chemical equilibrium, although the unburned gas is not.

Two classes of phenomena are found in nature which conform to the above model; these are called detonation waves and deflagration waves. (The term combustion wave covers both kinds of processes.) The qualitative differences between detonations and deflagrations are shown in Table 1.

TABLE 1 DIFFERENCES BETWEEN DETONATION AND DEFLAGRATION IN GASES

Ratio	Usual magnitude of ratio	
	Detonation	Deflagration
$\frac{v_u}{(\text{Acoustic vel})_u}$	5–10	0.0001–0.03
v_b/v_u	0.4–0.7	4–16
p_b/p_u	13–55	0.98–0.99999996
T_b/T_u	8–21	4–16
ρ_b/ρ_u	1.4–2.6	0.06–0.25

It is seen that, for a detonation, v_u/v_b, p_b/p_u, ρ_b/ρ_u, and $v_u/(\text{acoustic vel})_u$ are greater than unity, while, for a deflagration, they are less than unity (ρ is density).

The most striking feature of the combustion wave concerns the velocity v_u. For a given gas or mixture in a given thermodynamic state (T_u, p_u), there is found experimentally only one value of v_u for which a steady-state one-dimensional deflagration can exist, and only one value of v_u for which a steady-state one-dimensional detonation can exist. These values of v_u are generally referred to as the burning velocity and the detonation velocity, respectively. We note from Table 1 that the former is subsonic and the latter supersonic relative to the transition region.

Although this discussion is primarily concerned with deflagration, we shall first describe relationships sufficiently general to encompass detonation as well.

2 The Hugoniot Equation

Let us make a mathematical formulation of our problem according to standard aerodynamic procedure; i.e., we write

Received April 29, 1953.
[1] Research Engineer, Thermodynamics Section.

four relations: the conservation of mass, energy, and momentum, and the equation of state.

Conservation of mass:

$$v_u\rho_u = v_b\rho_b \quad \ldots\ldots [2.1]$$

Conservation of energy:

$$U_u + p_u/\rho_u + v_u^2/2 = U_b + p_b/\rho_b + v_b^2/2 \quad \ldots\ldots [2.2]$$

Conservation of momentum:

$$p_u - p_b = (v_u\rho_u)(v_b - v_u) \quad \ldots\ldots [2.3]$$

Equation of state:

$$p_u = \rho_u R T_u/m_u, \qquad p_b = \rho_b R T_b/m_b \quad \ldots\ldots [2.4]$$

U is internal energy per unit mass, m is molecular weight, R is the universal gas constant. We use an inertial system of units. An inviscid perfect gas is assumed.

Upon eliminating v_u and v_b from Equations [2.1], [2.2], and [2.3], we obtain the Hugoniot relation

$$U_b - U_u = (1/\rho_u - 1/\rho_b)(p_u + p_b)/2 \quad \ldots\ldots [2.5]$$

In any given case where the thermodynamic properties are known, values may be assigned at once to ρ_u, p_u, and U_u in Equation [2.5], and U_b in this equation may be considered to be a known (although often very complicated) function of T_b. But T_b is in turn a function of p_b and ρ_b (Equation [2.4]). Thus, the Hugoniot relation defines the locus of possible pairs of values of p_b and ρ_b in the p, ρ plane.

The Hugoniot relation is generally plotted as pressure vs. specific volume ($1/\rho$). Such a plot appears in Fig. 1. If the

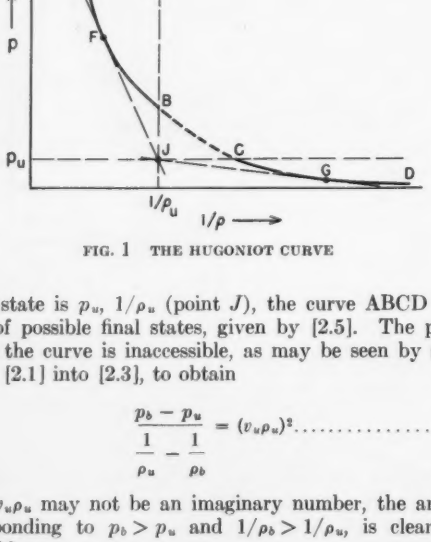


FIG. 1 THE HUGONIOT CURVE

initial state is p_u, $1/\rho_u$ (point J), the curve ABCD is the locus of possible final states, given by [2.5]. The portion BC of the curve is inaccessible, as may be seen by substituting [2.1] into [2.3], to obtain

$$\frac{p_b - p_u}{\frac{1}{\rho_u} - \frac{1}{\rho_b}} = (v_u\rho_u)^2 \quad \ldots\ldots [2.6]$$

Since $v_u\rho_u$ may not be an imaginary number, the arc BC, corresponding to $p_b > p_u$ and $1/\rho_b > 1/\rho_u$, is clearly inaccessible.

Thus, the value of $(p_b, 1/\rho_b)$ corresponding to the unique detonation velocity observed experimentally (for given initial conditions) must lie somewhere on arc AB, while the value corresponding to the unique deflagration velocity must lie somewhere on arc CD. Now, according to the Chapman-Jouguet hypothesis, the detonation point is simply F, the point of tangency of a straight line passing through the initial point J. This hypothesis may be justified theoretically by various arguments based on considerations of stability of the wave; see Courant and Friedrichs (1)[2] and Brinkley and Kirkwood (2). An impressive experimental justification may be made by calculating p_b and $1/\rho_b$ for a given mixture according to the above procedure (correcting for dissociative equilibria, by means of known thermodynamic properties of the burned gases), then obtaining the corresponding value of v_u from [2.6] and comparing with the measured detonation velocity. For example, for a mixture of two parts hydrogen and one part oxygen, Berets, Greene, and Kistiakowsky (3) calculate 2852 meter/sec and measure 2825–2833 meter/sec. Similar excellent agreement has been obtained for a variety of other mixtures.

The situation with regard to the deflagration point is by no means so straightforward. All that one can conclude without considering the internal structure of the wave is that the final deflagration state must lie somewhere between C and G on the Hugoniot curve. We shall see in later sections that the deflagration velocity (burning velocity) is controlled by the rates of chemical reactions and transport processes in the wave.

[2] Numbers in parentheses refer to References on page 354.

3 Burning Velocity: First Approximation

Let us consider the simplest conceivable model of a deflagration wave (Fig. 2). The chemical reaction is assumed

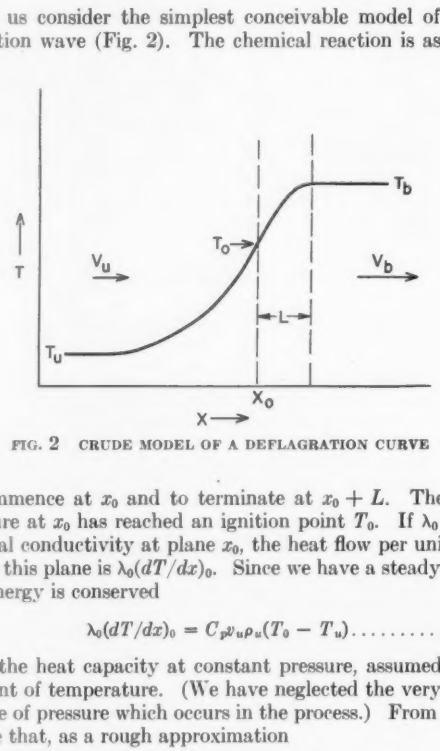


FIG. 2 CRUDE MODEL OF A DEFLAGRATION CURVE

to commence at x_0 and to terminate at $x_0 + L$. The temperature at x_0 has reached an ignition point T_0. If λ_0 is the thermal conductivity at plane x_0, the heat flow per unit area across this plane is $\lambda_0(dT/dx)_0$. Since we have a steady state, and energy is conserved

$$\lambda_0(dT/dx)_0 = C_p v_u \rho_u (T_0 - T_u) \quad \ldots\ldots [3.1]$$

C_p is the heat capacity at constant pressure, assumed independent of temperature. (We have neglected the very small change of pressure which occurs in the process.) From Fig. 2 we see that, as a rough approximation

$$(dT/dx)_0 \simeq 2(T_b - T_0)/L \quad \ldots\ldots [3.2]$$

From [3.1] and [3.2] we obtain what is essentially the Mallard-le Chatelier equation

$$v_u \simeq \frac{2\lambda_0}{C_p\rho_u L}\frac{T_b - T_0}{T_0 - T_u} \quad \ldots\ldots [3.3]$$

It is advantageous to replace the parameter L, the reaction zone thickness, by Q, the mean rate of heat release per unit volume in the reaction zone. L and Q are related by the over-all heat balance

$$QL = C_p v_u \rho_u (T_b - T_u) \quad \ldots\ldots [3.4]$$

Elimination of L between [3.3] and [3.4] yields

$$v_u \simeq \frac{1}{C_p\rho_u}\sqrt{\frac{2\lambda_0 Q}{T_b - T_u}\cdot\frac{T_b - T_0}{T_0 - T_u}} \quad \ldots\ldots [3.5]$$

Equation [3.5] permits interesting conclusions to be drawn concerning the effect of pressure on v_u, at constant T_u. To a fair approximation, C_p, λ_0, and T_b are independent of p, and ρ_u is directly proportional to p. Let us arbitrarily assume that T_0 is independent of p. The heat release rate Q will vary with p^2 for a second-order chemical reaction and with p for a first-order chemical reaction. From these assumptions, it follows from [3.5] that v_u is independent of p for a second-order reaction and inversely proportional to $\sqrt{p}$ for a first-order reaction. In spite of the crudeness of the model, these conclusions are essentially correct, as more refined treatments demonstrate.

Nevertheless, Equation [3.5] has a number of serious shortcomings. In the first place, no adequate independent means exists for evaluating the magnitudes of Q and T_0, and, hence, absolute magnitudes of v_u may not be predicted. In the second place, nothing can be concluded about the effect of T_u upon v_u, since the effect would be very sensitive to the value of T_0, which is unknown; the effect of T_u on Q is also unknown and can hardly be negligible. In the third place, we know that chemical reaction rates vary exponentially with temperature and depend on partial pressures of reactants, which in turn are influenced by diffusion; any theory which ignores these effects can hardly be complete.

4 More Rigorous Approach to Burning Velocity

In recent years, a number of advances have been made in the problem of formulating soluble flame equations which include detailed chemical kinetics and diffusion; the contributions of Lewis and von Elbe (1934), Zeldovich and Frank-Kamenetsky (1938), Boys and Corner (1949), and Hirschfelder and Curtiss (1949) are outstanding. These and many other studies are reviewed in detail by M. W. Evans (4). The treatment outlined in this section is patterned after that of Hirschfelder and Curtiss (5).

For simplicity, we treat the easiest possible case; a reactant molecule A changes exothermically into a product molecule B of the same molecular weight by a first-order process, the rate being given by

$$\frac{dy_B}{dt} = k y_A \exp(-E/RT) \quad [4.1]$$

Here, y_A and y_B are mole fractions ($y_A + y_B = 1$), t is time, and k and E are the frequency factor and activation energy of the reaction. We neglect the local variation of pressure in the flame. Since the mole fractions are governed by interdiffusion of A and B, expressions are formulated involving the diffusion velocities V_A and V_B. First we define v, the average flow velocity of the mixture at any point, by

$$v = v_u \rho_u / \rho \quad [4.2]$$

Then, we define V_A and V_B such that $(v + V_i)$ is the flow velocity of component i; i.e.,

$$v = y_A(v + V_A) + y_B(v + V_B) \quad [4.3]$$

from which it follows that

$$y_A V_A + y_B V_B = 0 \quad [4.4]$$

(Equations [4.3] and [4.4] are valid only if the molecular weights of A and B are the same.) Clearly, V_A is positive and V_B is negative in the transition zone. (Later, we shall see that V_A/v will be much greater than unity in part of the flame, but $|V_B/v|$ is never greater than unity.)

V_B is related to the mole fraction gradient (or partial pressure gradient, but not the concentration gradient) by a form of Fick's law

$$V_B = -\frac{D_{AB}}{y_B}\frac{dy_B}{dx} \quad [4.5]$$

where D_{AB} is the diffusion coefficient. (Equations for the V_i are much more complicated in multicomponent mixtures, or even in binary mixtures where the molecular weights of the components are not the same.)

It is now convenient to introduce a new variable, z_B, the fraction of the mass rate of flow due to B. By definition

$$z_B = \frac{y_B \rho (v + V_B)}{\rho_u v_u} \quad [4.6]$$

We eliminate V_B between [4.5] and [4.6], obtaining

$$\frac{dy_B}{dx} = \frac{v_u \rho_u}{D_{AB}\rho}(y_B - z_B) \quad [4.7]$$

Let us now rewrite [4.1] in terms of z_B. It is clear, from continuity of the species B, that

$$v_u \rho_u \frac{dz_B}{dx} = k\rho(1 - y_B)\exp(-E/RT) \quad [4.8]$$

Since we have three dependent variables, $z_B(x)$, $y_B(x)$, and $T(x)$, we need a third equation in addition to [4.7] and [4.8]. The new equation is based on the conservation of energy; we make an energy balance between plane x (in the transition zone) and plane b (in the completely burned gas, which is pure B)

$$\left[v_u\rho_u(1 - z_B)H_A + v_u\rho_u z_B H_B - \lambda\frac{dT}{dx}\right]_x = [v_u\rho_u H_B]_b \quad [4.9]$$

H_A and H_B are enthalpies per unit mass of A and B, respectively, and are considered to be known functions of T. The first and second terms in the left-hand bracket represent the energy carried across plane x (in the downstream direction) by species A and species B, respectively; the third term represents the heat conducted upstream across plane x. Thus the left-hand bracket, representing the net (downstream) energy flow across plane x, must equal the right-hand bracket, which is obviously the energy flow across plane b.

We may reduce the set of three equations [4.7], [4.8], and [4.9] to a set of only two equations by eliminating the variable x between the equations, since x only occurs as the differential dx. We then obtain

$$\frac{dz_B}{dT} = \frac{\lambda k\rho(1 - y_B)\exp(-E/RT)}{(v_u\rho_u)^2[(1 - z_B)H_A + z_B H_B - (H_B)_b]} \quad [4.10]$$

and

$$\frac{dy_B}{dT} = \frac{\lambda(y_B - z_B)}{D_{AB}\rho[(1 - z_B)H_A + z_B H_B - (H_B)_b]} \quad [4.11]$$

Before solving [4.10] and [4.11], one must express λ, D_{AB}, ρ, H_A, and H_B as functions of T. Then, one formulates the hot and cold boundary conditions of the flame and seeks functions $z_B(T)$ and $y_B(T)$ which will satisfy [4.10], [4.11], and the boundary conditions. It will turn out that, once values are assigned to the parameters k, E, λ_u, $(D_{AB})_u$, ρ_u, $(H_A)_u$, and $(H_B)_u$, there will only be a single value of v_u (an eigenvalue) for which all conditions can be satisfied. This, then, is the burning velocity corresponding to this model and this assignment of parameters.

The hot boundary condition of our simple model is

$$z_B = y_B = 1 \qquad \text{when } T = T_b \quad [4.12]$$

The cold boundary condition introduces special difficulties, however, because the exponential rate law gives a small but *finite* reaction rate, proportional to $\exp(-E/RT_u)$, in the unburned gas. Since $T = T_u$ only at $x =$ minus infinity, and since the gas flows at a finite velocity in the region between $x =$ minus infinity and $x =$ some finite value corresponding to the inflection point of the $T(x)$ curve, one can conclude that the gas has already completely reacted far up-

stream from the transition zone. This is clearly unreasonable. Therefore, it is logically inconsistent to specify a cold boundary condition of the form

$$z_B = y_B = 0 \qquad \text{when } T = T_u \ldots\ldots\ldots [4.13]$$

The model must be modified in some way to meet this objection. Hirschfelder, et al., have located an artificial heat sink at a large but finite distance upstream from the inflection point, and they obtain a cold boundary condition by assuming a small, arbitrary heat flux into the sink. A quite different approach is that of Friedman and Burke (6), who use a term of the form

$$\exp\left[-\frac{E}{RT_b}\cdot\frac{T_b - T_u}{T - T_u}\right] \ldots\ldots\ldots\ldots [4.14]$$

in Equation [4.8], so that the reaction rate is zero in the unburned gas. In this case, [4.13] may be used as a cold boundary condition, and no heat sink is needed.

Friedman and Burke made numerical integrations of the pair of differential equations [4.10] and [4.11] for a range of values of the parameters, by means of an IBM Card-Programmed Electronic Calculator. They used [4.14], assumed C_p and k to be independent of T, and took λ, D_{AB}, and ρ to be proportional to T, T^2, and T^{-1}, respectively. They obtained 12 eigenvalues of v_u corresponding to 12 sets of values of the pertinent dimensionless parameters, and found that the results could be represented empirically by

$$\beta = \left[\frac{0.514}{\alpha}\epsilon^2 + \left(1 + \frac{0.35}{\alpha}\right)\epsilon + \left(0.20 + \frac{0.05}{\alpha}\right)\right]\exp(\epsilon) \ldots [4.15]$$

Here

$\beta = \kappa_u k/v_u^2$ = parameter containing burning velocity
$\kappa = \lambda/C_p\rho$ = thermal diffusivity of gas
$\epsilon = E/RT_b$ = dimensionless activation energy
$\alpha = \kappa/D_{AB}$ = Lewis number (7) of gas

Equation [4.15] is valid for $2 < \epsilon < 22$, and $0.5 < \alpha <$ infinity. (According to the kinetic theory of gases, α is of the magnitude of unity.) Study of Equation [4.15] shows that the dependence of β upon α increases markedly with increasing ϵ. A physical interpretation of this behavior has been made (6). Of course, this solution describes only the extremely simple model $A \rightarrow B$, with the additional assumptions noted above. Generalized extensions of this approach to more complicated models (e.g., flames involving sequences of chemical reactions) have not yet been carried out, although the formal approach to such computations has been outlined by Hirschfelder, et al.

Before leaving this section, we shall find it instructive to consider Figs. 3 and 4, which were calculated from one of the

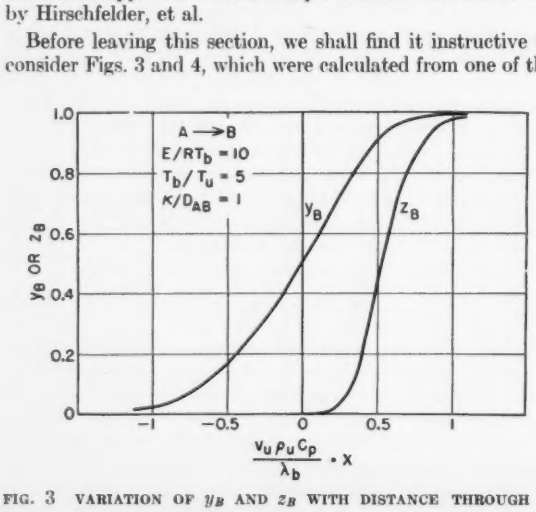


FIG. 3 VARIATION OF y_B AND z_B WITH DISTANCE THROUGH A FLAME

numerical integrations ($\epsilon = 10$, $\alpha = 1$) of the simple model $A \rightarrow B$. From Fig. 3, we see the great difference between $y_B(x)$ and $z_B(x)$. (In this plot, x is arbitrarily measured from the plane at which half the temperature rise has occurred.) From the $z_B(x)$ curve we find that the maximum reaction rate occurs when $(T - T_u)/(T_b - T_u) = 0.91$, and only one per cent of the reaction has occurred when $(T - T_u)/(T_b - T_u) = 0.67$. On the other hand, the $y_B(x)$ curve tells us that, at the one per cent reaction point, the gas mixture already consists of two thirds B and one third A! This large quantity of B in the prereaction zone maintains itself by diffusion, of course. In Fig. 4 we see V_A/v and V_B/v plotted vs. $(T - T_u)/(T_b - T_u)$, and we observe that in

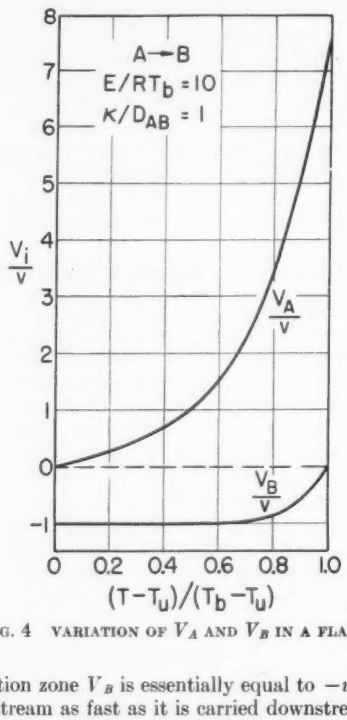


FIG. 4 VARIATION OF V_A AND V_B IN A FLAME

the prereaction zone V_B is essentially equal to $-v$, so that B diffuses upstream as fast as it is carried downstream, in this region. We also note from Fig. 4 that V_A/v may be as large as seven; this effect would be of great importance if we were trying to estimate the residence time of A in the flame.

5 Experimental Aspects of Burning Velocity

The combustible gases of perhaps most general interest are hydrocarbon vapor-air mixtures. Development of the theory to account for the burning velocities of such mixtures has not yet progressed very far. The first step in this direction is the accurate measurement of burning velocity as a function of pressure, initial temperature, and fuel-air ratio, for certain representative hydrocarbons. Then, we compare with theoretical predictions. Such comparisons will be illustrated.

Some very recent measurements by Johnston and Cyphers (8) over a tenfold range of pressure show that burning velocity varies with an inverse power of pressure equal to 0.25 for 2,2,4-trimethylpentane-air, 0.19 for propane-air, and 0.18 for benzene-air. (In each case, the mixture of maximum burning velocity was studied.) A theoretical analysis such as that in the previous section shows that burning velocity should vary inversely with the 0.50 power of pressure for a first-order chemical reaction and should be independent of pressure for a second-order reaction (or sequence of second-order reactions). It is therefore strongly indicated that hydrocarbon-air flames involve some combination of first-order and second-order reactions, and that no theory based on a one-step

reaction mechanism could adequately describe the process. Of course, the above argument does not rule out the possibility of third-order as well as first-order and second-order reactions in these flames.

Dugger and Heimel (9), among others, have measured the variation of burning velocity with initial temperature. By comparing these results with theories based on one-step reaction mechanisms, one is able to obtain apparent values of the activation energy E. For a propane-air mixture, their data lead to a value of about 37 kcal per mole for E when a first-order reaction is assumed, and about 27 kcal per mole when a second-order reaction is assumed. If one considers this to be a complex reaction involving both first-order and second-order steps, then very little can be concluded from these measurements concerning the activation energies of the individual steps. For example, the Rice-Herzfeld treatment (10) of isothermal chain reactions shows that the apparent activation energy of the over-all reaction may be considerably less than the activation energy of the slowest step.

In view of the apparent kinetic complexity of hydrocarbon-air systems, the desirability of seeking simpler systems for study is obvious. Furthermore, isothermal reaction rate studies of certain systems (but not hydrocarbon-air) at relatively low temperatures (200–600 C) have led to tentative detailed mechanisms, with numerical values for the activation energies and frequency factors. Only limited burning velocity studies of such systems have been made as yet. Table 2 shows a list of pure gases either known or strongly

TABLE 2 SOME DECOMPOSITION FLAMES

Substance	Main reaction	ΔH^a
Ozone	$O_3 \rightarrow {}^3/_2\, O_2$	−34
Ethylene oxide	$C_2H_4O \rightarrow CO + CH_4$	−32
Hydrazine	$N_2H_4 \rightarrow {}^1/_3\, NH_3 + {}^1/_3\, N_2$	−37
Azomethane	$CH_3N_2CH_3 \rightarrow C_2H_6 + N_2$	−50
Nitric oxide	$NO \rightarrow {}^1/_2\, N_2 + {}^1/_2\, O_2$	−22
Hydrogen peroxide	$H_2O_2 \rightarrow H_2O + {}^1/_2\, O_2$	−25
Cyanogen	$C_2N_2 \rightarrow 2C_{(s)} + N_2$	−74
Acetylene	$C_2H_2 \rightarrow 2C_{(s)} + H_2$	−54
Nitromethane	$CH_3NO_2 \rightarrow NO, CH_4, CO, H_2O$	?

[a] ΔH = heat of reaction (kcal/mole gaseous reactant)

suspected to be capable of supporting a deflagration in the complete absence of any other reactant. It is probably true that any gas capable of decomposing with the liberation of 25 or 30 kcal per mole can support a deflagration, and, if somewhat more energy is available, a detonation can be supported as well. Certain of these decomposition flames (ethylene oxide, nitromethane, azomethane) correspond approximately to first-order kinetics when the gases are studied isothermally at low temperatures, so it would be quite interesting to measure the pressure dependence of burning velocity. Friedman and Burke are now studying ethylene oxide from this point of view (11).

Table 3 shows some combustible systems which are more complex than decomposition flames in the sense that two reactants instead of one are involved, but are simple (compared with hydrocarbon-oxygen flames) in that only two chemical elements are involved in each case. We note from the table that the burning velocity of $2CO + O_2$ is far smaller than that of $2H_2 + O_2$, even though T_b is about the same for

TABLE 3 TWO-ELEMENT TWO-COMPONENT FLAMES

Reaction	Burning velocity, cm/sec	Flame temperature, T_b, ° K
$H_2 + F_2 \rightarrow 2HF$	..	4300
$2H_2 + O_2 \rightarrow 2H_2O$	1100	3000
$H_2 + Cl_2 \rightarrow 2HCl$	400	2500
$H_2 + Br_2 \rightarrow 2HBr$	30	1600
$2CO + O_2 \rightarrow 2CO_2$	25	3000

the two mixtures. This leads us to suspect that the chemical mechanisms must be quite different in these two cases Furthermore, it is found that addition of 1 per cent water vapor to $2CO + O_2$ will double the burning velocity. Tanford and Pease (12) have attributed this behavior to atomic hydrogen which forms in the burned gas from H_2O in accordance with dissociation equilibria, and which then diffuses back into the reaction zone, initiating chain reactions. Attempts have been made (13, 14, 15) to show that the Tanford-Pease mechanism determines the rate-limiting step for hydrocarbon-air flames, without entirely conclusive results. Further studies of the effect of water vapor on burning velocity of nonhydrogen-containing systems (O_3, CS_2–O_2, CO–Cl_2, C_2N_2–O_2) would appear to be desirable.

A considerable body of kinetic information is available about the chain reactions involved in the flames of Table 3. It seems likely that attempts will soon be made to relate this information to flame behavior.

6 Structure of Combustion Waves—Experimental

We have already seen how $T(x)$, $y_i(x)$, and $z_i(x)$ may be calculated for a deflagration wave, given a few measured values of v_u as a function of T_u and p. We assume that the chemical process is a single reaction of simple order, we determine the order from the variation of v_u with p, and then we evaluate the kinetic parameters k and E from the variation of v_u with T_u. The other parameters (λ, D, C_p, T_b) are, in principle, measurable in separate experiments. From this point it is simply a matter of numerical integration to obtain the desired relations.

However, since chain reactions may be occurring which involve intermediate species whose very existence we have ignored in our model, we can expect that the calculated $T(x)$ will only be a first approximation, while $y_i(x)$ and $z_i(x)$ may be altogether incorrect. Therefore, direct measurements of temperature and partial pressure patterns in a deflagration wave would be of interest.

Optical, mass, and even microwave spectroscopy have been used to obtain valuable bits of qualitative information concerning intermediate species in flames. Gaydon (16) has been particularly active in this work. Still, progress to date has not been sufficient to yield $y_i(x)$ curves for all species in even the simplest of flames. The situation with regard to $T(x)$ curves is somewhat further advanced, and will be discussed in some detail.

Before discussing techniques for measuring $T(x)$, one must consider whether it is meaningful to speak of a temperature in the reaction zone. First, we forget about internal degrees of freedom and ask whether a translational temperature always exists in the reaction zone; i.e., in each layer, is the distribution of molecular velocities approximately in accord with the Maxwell-Boltzmann law? Rough calculations (17) show that the thickness of the zone of intense chemical reaction for a deflagration is of the order of magnitude of the ratio of the mean free path to the Mach number. For relatively slow-burning flames ($v_u \sim 40$ cm/sec), then, the reaction zone comes out to be $\sim$1000 mean free paths in thickness; hence there is very little possibility of abnormal molecular velocity distributions. For very fast flames ($v_u \sim 1000$ cm/sec), the reaction zone may be only 20 or 30 mean free paths in thickness, and some slight departure from the Maxwell-Boltzmann distribution is conceivable.

Now, we consider internal degrees of freedom. There are two effects. First, there is the well-known time lag (18) in the excitation of vibrational modes of molecules, which would tend to cause the vibrational temperature to be lower than the translational temperature in the zone of rapid temperature rise. Second, there is the possibility that newly formed molecules in the reaction zone are created in electronically, rotationally, or vibrationally excited states (19), so these

molecules may temporarily have abnormally high internal temperatures. Spectroscopic studies (16) indicate that this actually happens, but Penner (20) has pointed out that the observations may have been misinterpreted, and the anomalously high internal temperatures (several times as high as T_b) that have been reported may not actually exist.

Three techniques have been proposed quite recently for measuring the pattern of translational temperature in a flame. Andersen and Fein (21) and Fristrom, et al. (22) have measured the curving streamlines through a flame inclined at an angle to the direction of initial flow. Dust particles are used as tracers. Analysis of the streamlines, with suitable assumptions, leads to a temperature profile. Dixon-Lewis and Wilson (23) and Burgoyne and Weinberg (24) have used optical techniques to obtain the variation of refractive index through a flat flame. Klaukens and Wolfhard (25) and Friedman (26) have used fine thermocouples to traverse a flame. None of the work reported to date has been refined to the point of giving a completely reliable detailed picture of $T(x)$, but refinements are possible and more thorough studies will undoubtedly be reported in the near future. Hence, no comment will be made on the tentative results now available.

7 Stability of Deflagration Waves

Up to now, we have confined our discussion to purely one-dimensional waves, without considering whether such waves would be stable. Experimental studies reveal several kinds of instability: (a) transition from deflagration to detonation; (b) transition from a flat to a "cellular" deflagration wave, or from an axially symmetrical conelike flame to a "ribbed" flame; (c) quenching of the flame by the walls of the vessel, the effect becoming prominent when the vessel is sufficiently small or the pressure is sufficiently low; (d) transition to turbulence. We shall only make a few brief comments about each of these effects.

Let us consider this question: What determines whether a given combustible mixture will burn as a detonation or as a deflagration? It is found experimentally that some mixtures will burn in either mode, while other mixtures can only deflagrate. For the former mixtures, deflagration is obtained when the burned gases are unconfined, while sufficient confinement of the burned gases produces a transition to detonation. The latter mixtures, which cannot detonate, are characterized by low burning velocity. A glance at data for rich and lean detonation limits and the corresponding burning velocities shows that the critical ratio of burning velocity to acoustic velocity in the unburned gas is about 0.0012–0.0015. If the ratio is below this, the gas will not detonate. Of course, the empirical nature of this criterion must be kept in mind, and it seems possible that normally nondetonating mixtures could be detonated by employing larger and longer tubes or higher initial pressures than those customarily employed in experiments of this type.

Turning now to "cellular" and "ribbed" flames, we find (27, 28) that these are only encountered when the reactant gas consists of at least two types of molecules of considerably differing molecular weight, the component of higher molecular weight being present in excess (relative to the mixture of maximum burning velocity). For example, quite rich C_4H_{10}-O_2 mixtures and quite lean H_2-O_2 mixtures exhibit "cellular" behavior, the "cells" being more or less hemispherical and convex toward the unburned gas. This configuration appears to be stabilized by preferential diffusion, which would tend to enrich the unburned mixture in the heavier component, in the region between cells.

The quenching distance X (e.g., distance between cold parallel walls which will just permit flame propagation), to the first approximation is given by

$$X = C_1\kappa_u/v_u \quad \ldots\ldots\ldots\ldots\ldots\ldots [7.1]$$

However, C_1, a dimensionless constant of the order of magnitude of 50, varies considerably from mixture to mixture. The writer has pointed out (29) that the quenching distance X is of the same magnitude as the characteristic size of the cells of cellular flames, thus showing the intimate relation between the two phenomena. Finally, in connection with quenching, it may be noted that Friedman and Johnston's data (30) show that the "pressure limit of inflammability" for a given mixture, usually measured in tubes, is the same as the pressure corresponding to quenching by the tube.

There are two outstanding problems in connection with turbulent flames. First, it is necessary to distinguish behavior due to fully developed turbulence from behavior associated with a laminar-turbulent transition region. Second, a distinction must be drawn between effects due to flame-generated turbulence and to turbulence originally present in the unburned gases.

These various departures from one-dimensional behavior are discussed extensively in review papers by Markstein, Wohl, and Karlovitz (31). Reference may also be made to the treatises of Lewis and von Elbe (32) and Gaydon and Wolfhard (33).

Since this paper was written, two quite significant publications in this field have appeared. These are: "The Theory of Flame Propagation. IV," by J. O. Hirschfelder, C. F. Curtiss, and D. E. Campbell, *Journal of Physical Chemistry*, vol. 57, 1953, pp. 403–414; and "The Effect of Water on the Burning Velocities of Cyanogen-Oxygen-Argon Mixtures," by R. S. Brokaw and R. N. Pease, *Journal of the American Chemical Society*, vol. 75, 1953, pp. 1454–1457.

References

1 "Supersonic Flow and Shock Waves," by R. Courant and K. O. Friedrichs, Interscience, New York, 1948.

2 "On the Condition of Stability of the Plane Detonation Wave," by S. R. Brinkley, Jr., and J. G. Kirkwood, Third Symposium on Combustion, Williams and Wilkins, Baltimore, 1949, pp. 586–594.

3 "Gaseous Detonations. I. Stationary Waves in Hydrogen-Oxygen Mixtures," by D. J. Berets, E. F. Greene, and G. B. Kistiakowsky, *Journal of the American Chemical Society*, vol. 72, 1950, pp. 1080–1086.

4 "Current Theoretical Concepts of Steady-State Flame Propagation," by M. W. Evans, Chemical Reviews, vol. 51, 1952, pp. 363–429.

5 "The Theory of Flame Propagation," by J. O. Hirschfelder and C. F. Curtiss, *Journal of Chemical Physics*, vol. 17, 1949, pp. 1076–1081. Cf. also Third Symposium on Combustion, Williams and Wilkins, Baltimore, 1949, pp. 121–140; *Journal of Physical and Colloid Chemistry*, vol. 55, 1951, pp. 774–788; Fourth Symposium on Combustion, Williams and Wilkins, Baltimore, 1953, pp. 190–211.

6 "A Theoretical Model of a Gaseous Combustion Wave Governed by a First-Order Reaction," by R. Friedman and E. Burke, *Journal of Chemical Physics*, vol. 21, 1953, pp. 710–714.

7 "The Evaporation of a Liquid into a Gas," by W. K. Lewis, *Mechanical Engineering*, vol. 44, 1922, pp. 445–446.

8 W. C. Johnston and J. A. Cyphers, Westinghouse Research Laboratories, private communication, February 1953.

9 "Flame Speeds of Methane-Air, Propane-Air, and Ethylene-Air Mixtures at Low Initial Temperatures," by G. L. Dugger and S. Heimel, *NACA Technical Note* 2624, February 1952.

10 "The Thermal Decomposition of Organic Compounds from the Standpoint of Free Radicals. VI. The Mechanism of Some Chain Reactions," by F. O. Rice and K. F. Herzfeld, *Journal of the American Chemical Society*, vol. 56, 1934, pp. 284–289.

11 "Burning Velocity of Ethylene Oxide Decomposition Flames," by R. Friedman and E. Burke, *Journal of Chemical Physics*, vol. 21, 1953, p. 165.

12 "Theory of Burning Velocity," by C. Tanford and R. N. Pease, idem., vol. 15, 1947, pp. 431–439, 861–865.

13 "Burning Velocities of Butadiene-1,3 with Nitrogen-Oxygen and Helium-Oxygen Mixtures," by E. J. Badin, J. G. Stuart, and R. N. Pease, idem., vol. 17, 1949, pp. 314–316.

(Continued on page 374)

Determination of Thrust Alignment in Rocket Engines

W. L. ROGERS[1]

Aerojet-General Corporation, Azusa, Calif.

As a result of deviations in the flight path of early missiles during launching, a program of thrust-alignment testing of full-scale rocket motors was initiated at the Aerojet-General Corporation. Design and construction of the test facility are described. Procedures used and problems encountered are outlined, and methods used in reduction of data explained. Data from representative tests are presented.

IN THE modern theory of rocket flight the possibility of a deviation between the geometric center line of a rocket nozzle and the true position of the line of thrust of the rocket is recognized. The source of deviation, called "gas misalignment," is discussed by Rosser, et al.,[2] and the need for a systematic study is expressed.

Attempts had been made to measure the actual position of the line of thrust from rocket engines, but without real success, principally because of the lack of adequate equipment. The equipment must measure accurately very small forces in planes normal to the plane of a very large force; in a solid propellant rocket these forces are applied by a body whose mass is reduced during the observations.

The problem was attacked by the design and construction of a facility to measure thrust alignment in large solid propellant rockets. The basic requirement for this facility was the ability to measure with reasonable accuracy thrust-line angles of 0.05 deg (three minutes) and thrust-line displacement of $^1/_{32}$ in.

Six components of force are required to define completely the magnitude and position of a force in space. The question first to be answered in design of a thrust-alignment test setup is the arrangement of these six reactions to be used. Three arrangements were considered (Fig. 1).

1 The rocket motor is mounted horizontally, as in a conventional static firing, with the thrust reaction and three other reactions horizontal and the remaining two vertical.

2 The rocket motor is mounted vertically with the thrust distributed among three vertical reactions, and the three other reactions in a horizontal plane.

3 The rocket motor is mounted vertically, with the thrust reaction in a single force, the remaining five reactions being distributed horizontally in two planes.

The primary criterion for selection of the system was the accuracy to be expected in the measurement of thrust deviations. On this basis the first arrangement can be eliminated immediately, since the weight of the rocket motor must be subtracted from the vertical reactions to obtain the force normal to the rocket center line in the vertical plane, and since the rocket motor weight changes continuously during the observation at a rate which cannot be established closer than the same order of magnitude of the forces to be measured in the vertical plane.

The second and third arrangements overcome this disadvantage; the gages which measure the thrust force also measure the weight of the rocket. Since the weight change during firing is small in comparison with the thrust (approximately 1 per cent solid propellant booster rockets tested), quite accurate results can be obtained even though the rate of change in weight cannot be measured with great accuracy.

In choosing between arrangements 2 and 3 it was recognized that arrangement 2 is more economical of space, since the gages need not extend beyond the end of the motor. In addition, the largest gage is one third the size of the gage required in arrangement 3, since the thrust is distributed among three gages. However, these advantages do not outweigh the main disadvantage of arrangement 2, which is not present in arrangement 3. In arrangement 2 the measurement of moments about an axis normal to the center line of the rocket depends upon differences in the loads measured by the three thrust-reaction gages. Since these moments are expected to be small, it is clear that greater accuracy is to be expected from measurement of these moments by the horizontal gages of arrangement 3, which measure only side forces. Actually, in the study made before final selection of the arrangement to be used, it was found that for the structure under consideration the inherent error in determination of the position of the thrust line (by measurement of the moment referred to above) was 0.066 in. for arrangement 2, as compared with 0.011 in. for arrangement 3.

The next problem considered was the design of the force-measuring members to insure maximum accuracy. Because of the unusual range of forces to be measured it was necessary to insure that friction would not influence the results. Also, because of the high force applied, it was necessary to

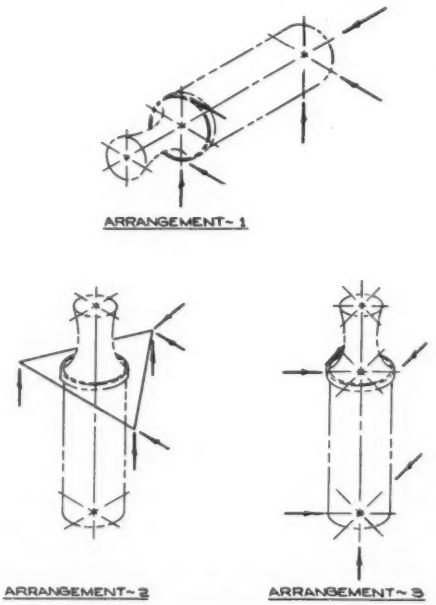


FIG. 1 POSSIBLE ARRANGEMENTS FOR THRUST ALIGNMENT TESTING

Received January 19, 1953.

[1] Assistant General Manager. Member ARS.

[2] "The Mathematical Theory of Rocket Flight," New York; McGraw-Hill Book Co., Inc., 1947, chapter III, p. 179.

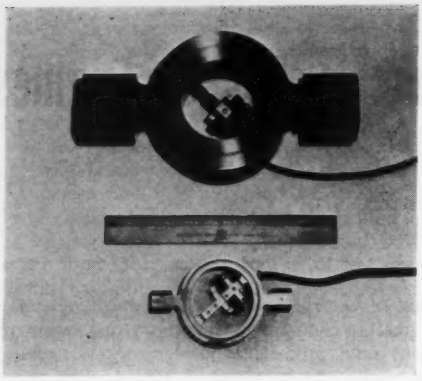


FIG. 2 FORCE PICKUP

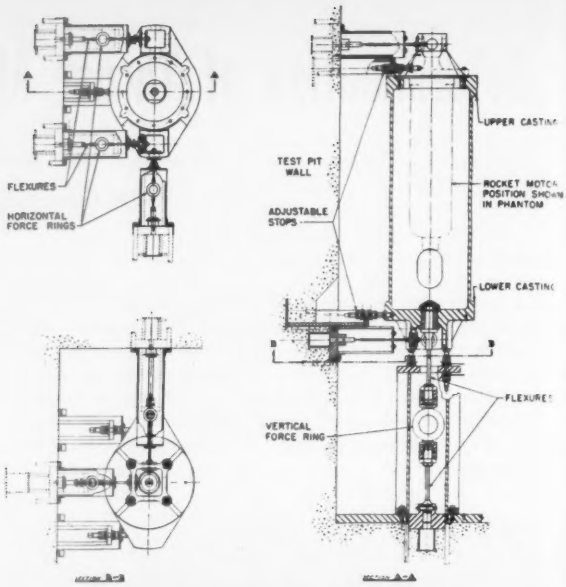


FIG. 3 THRUST ALIGNMENT FIXTURE DESIGN

mount the rocket motor securely. These requirements suggested the use of flexure members as hinges. Using flexure members the deflections under load must be limited so as not to exceed the elastic limit of the flexure hinge. The force ring limits deflection, since it is manufactured in many sizes, all showing a deflection of 0.005 in. under rated load. This force-measuring device (Fig. 2) is so designed that the elastic element is separated from the electrical element, permitting complete freedom of movement for the elastic element. The ring is machined from a single piece of metal so that internal and external bosses are integral parts. The magnetic core and armature are so mounted that the relative motion is twice the motion of the external connections.

The design of the thrust-alignment fixture is shown in Fig. 3. It consists of a large steel tube with castings attached at either end. The lower casting provides for attachment of the vertical force-ring flexure and the two lower horizontal force-ring flexures. The upper casting provides a carefully machined mating face for installation of the rocket, and bosses for attachment of the three top force-ring flexures. All flexure attachments to the thrust-alignment fixture and to the test-pit walls are made through close-fit holes and pins. The fixture is first placed within the test pit, carefully located, and clamped in place. The holes for the flexure in the test-pit walls are then drilled or bored, and reamed using the mating holes in the fixture as a locating jig. Also provided are adjustable stops to prevent damage to the gages from unanticipated loads, and covers to prevent damage to the gages in the event of an explosion.

The test pit itself (Fig. 4) is of reinforced concrete with walls 2 ft thick. It is 30 ft high and is topped by a traveling hoist which can be moved outside the back wall of the pit during firing. A blast plate is provided at the top of the pit.

The fixture shown in Fig. 3 was designed to accommodate units of various sizes. It was used for two series of tests on solid-propellant booster rockets and one series of tests on a liquid-propellant rocket engine. A requirement for testing larger solid-propellant booster rockets with a canted nozzle led to the construction of a larger fixture, similar to that shown in Fig. 3.

A schematic diagram of the electrical circuit used in these measurements is shown in Fig. 5. A force ring deflects under the external load, increasing one air gap between the magnetic core and the armature and decreasing the other. This deflection changes the flux, since the reluctance of the two air gaps is changed. Thus the current flowing in the two coils is changed. The discriminator coupling unit applies a 3000-cycle carrier frequency to the two coils on the ring and translates the output into a d-c signal which deflects the galvanometer in the recording oscillograph. The movement of the galvanometer and attached mirror is translated as a trace on photosensitive paper by means of a light source which is

FIG. 4 THRUST ALIGNMENT TEST PIT SHOWING (1) CALIBRATION BEAMS, (2) CRANE HOIST, (3) BLAST PLATE, AND (4) STEEL PADS EMBEDDED IN CONCRETE

reflected by the mirror. The oscillograph is provided with a means of checking the sensitivity of each galvanometer before and after firing by applying a known voltage to the galvanometer and recording deflection. This deflection is compared with that obtained during calibration, to insure that comparable values are obtained.

All thrust-alignment tests are preceded and followed by calibration of the setup. Calibration loads are applied by dead weights through bell cranks, or, with large loads, through standardized hydraulic jacks.

The calibration load which simulates the thrust force of the rocket engine is applied from the top of the test pit through a long column which bears upon a steel ball accurately located in a plate placed on the flange which holds the rocket motor during test. The vertical calibrating load is transmitted to

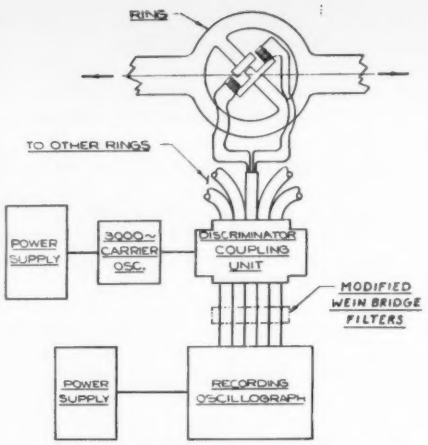


FIG. 5 THRUST ALIGNMENT SETUP. ELECTRICAL CIRCUIT FOR FORCE MEASUREMENT

the test-pit walls through two beams which span the pit at the top.

When calibration is completed, the motor is lowered into place in the fixture. In order to obtain a reference axis to which to relate the measured line of thrust, the rocket nozzle has been previously attached to an adapter plate and the position of the nozzle axis with respect to the outside diameter and mating face of this plate carefully measured. The adapter plate is then bolted to the machined face in the upper casting of the thrust-alignment fixture. The diameter of the adapter plate is machined slightly greater than that of the shoulder on the fixture, and the plate is shrunk into place to preclude errors in lateral positioning. The rocket motor in place in the test fixture is shown in Fig. 6.

Using the equipment described above, data can be obtained from which the line of thrust of a rocket engine can be established. However, additional work is required to translate the data into usable form. Fig. 7 shows the data obtained on one of the tests in the first series. The test fixture is essentially a mass supported on six springs, so that when the thrust load is applied the system is set into vibration. Since the natural frequencies of the system (less than 60 cps) are considerably lower than the frequency of response of the recording apparatus (200 cps), the gages can record the resulting oscillations. No attempt was made to eliminate or reduce these oscillations prior to the first series of firings, as it was desired to make sure that the rocket thru t characteristic did not include frequencies within the measuring capacity of the instrumentation.

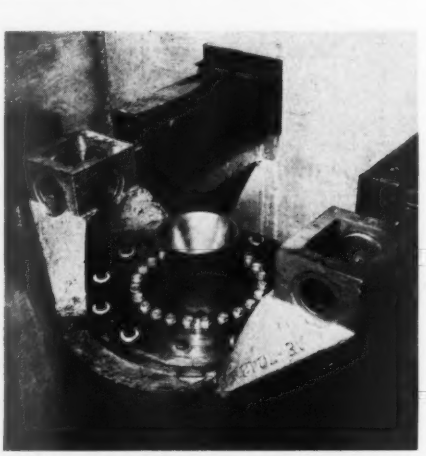


FIG. 6 ROCKET MOTOR IN PLACE IN TEST FIXTURE

This point was checked by comparing the frequency of oscillation of the fixture in the few hundredths of a second immediately after the igniter has fired (but before the nozzle closure has burst), with the frequency of oscillation after the initiation of thrust. Another check was made by comparing the oscillation frequency near the end of the test with the frequency after the cessation of thrust. In both comparisons the frequencies were found to be the same. After the comparisons had been made, however, it was desirable to eliminate these oscillations, to facilitate the reduction of data and to permit use of greater galvanometer deflection. With the oscillations present, it was not uncommon to measure a force whose magnitude was 5 per cent of the maximum oscillation. If the oscillations could be eliminated, the galvanometer deflection could be increased five to twenty times to record the same force, so that considerably better accuracy in reading the data could be expected.

The original data, which included the large oscillations in all traces, were reduced by reading ordinates on each trace at 0.01-sec intervals, correcting the values from calibration charts, averaging the ordinates over a 0.2-sec base period using Simpson's rule, and plotting the resultant points. In order to reduce scatter in the final plotted points, base periods were overlapped by 0.05 sec.

In this method of reducing data there is some uncertainty as to the instantaneous thrust values within the base periods used, particularly at the very beginning of thrust. In order to insure that significant side forces were not overlooked, the possible effects of side forces acting for these short times were investigated. In a typical missile, for action times less than 0.10 sec, it was found that a side force in excess of 150 lb would be required to produce 0.25-deg yaw of the missile. The average values of side forces obtained in thrust-alignment tests during the first 0.2 sec were from 10 to 20 lb.

Elimination of the oscillations to simplify data reduction can theoretically be accomplished either electrically or mechanically. However, before a method for elimination of the oscillations was selected, a harmonic analysis was obtained for a typical record of the main thrust pickup and for one of the side force pickups in order to obtain better understanding of the basic frequencies of which these oscillations were composed.

Data from a typical harmonic analysis of an unfiltered X_1 force trace are shown in Fig. 8. The five different curves plotted are based on different assumptions as to wave length and time interval used as the basis of the analysis. Three significant frequencies appear in the trace. The largest-am-

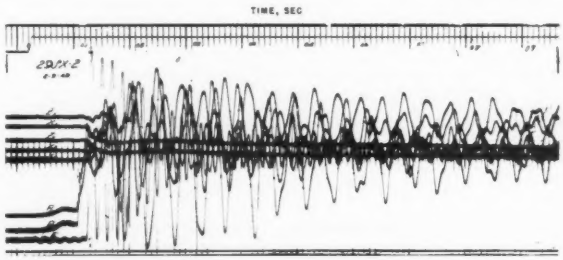


FIG. 7 REPRESENTATIVE RECORD OF FIRST THREE FIRINGS

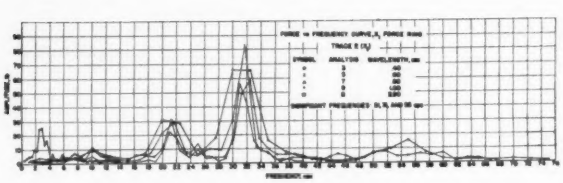


FIG. 8 DATA FROM A TYPICAL HARMONIC ANALYSIS OF AN UNFILTERED X_1 FORCE TRACE

plitude frequency, 31 cps, is the characteristic natural frequency of the X_1 suspension. The 55-cps frequency is known to be the natural frequency of the vertical suspension; its appearance indicates the coupling between the two suspensions. The 21-cps frequency and, of lesser significance, the 10-cps frequency, represent coupling of the X_3 suspension and the torsional mode, respectively, about the vertical axis. The existence of the torsional mode is confirmed by analysis of the X_2 suspension, which shows the same component.

A similar analysis conducted on the same trace after the cessation of burning disclosed significant components of frequency only at 31 and 21 cps. It could be concluded from this analysis that the 55-cps and 10-cps components were imposed by the thrust force. However, it is known that the 55-cps component results from the natural period of the vertical suspension, and the 10-cps component from the torsional mode. Thus, it is concluded that these components, which were shown in Fig. 8 to have a low coupling coefficient, were merely damped out subsequent to the end of burning.

By installing suitable filters to eliminate these four frequency components, an almost completely smooth trace could be obtained. Actually, a single filter at the characteristic natural frequency of the force-ring support provided adequate in all tests.

Because of the low inherent damping and large ratio of mass to spring constant in the mechanical system, the addition of mechanical damping is difficult. The physical proportions of a vane-oil-damping system would cause a severe installation problem.

If the frequency characteristic of the recording system exhibits an amplitude null point coincident with the natural frequency of the force-measuring system, the main oscillation can be removed from each trace. A modified Wein bridge-type filter, having a single rejection frequency corresponding to the particular mode natural frequency, was selected for this purpose and interposed between the oscillograph and coupling unit. Fig. 9 shows a representative record of a firing using the Wein bridge-type filters. Reduction of data from records of this type is considerably simpler than reduction of data from records such as shown in Fig. 7.

Discussion of some results obtained in the thrust-alignment facilities follows. Typical reduced data will be presented and comments will be made upon the significance of these results. Fig. 10 shows graphically the co-ordinates used to evaluate the position of the line of thrust.

Series 1

This series of tests was performed to investigate the effects of fabrication tolerances on the line of thrust of a rocket engine with a view toward reducing the cost of nozzle fabrication. The series consisted of the following tests:

1 Control firing—made with a normal nozzle utilizing a 100-micron machine finish on the interior of the throat and exit sections.

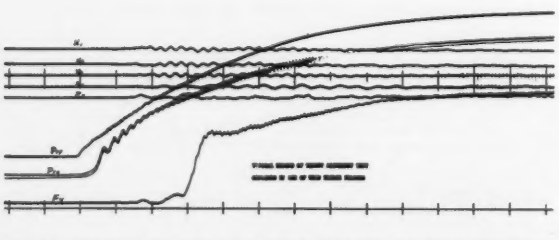


FIG. 9 TYPICAL RECORD OF THRUST ALIGNMENT TEST OBTAINED BY USE OF WEIN BRIDGE FILTERS

2 Nozzle throat contour—same as Test 1 except that the radius of the nozzle throat section was larger (2.50 in. as compared with 1.38 in.).

3 Nozzle throat surface finish—same as Test 2 except that the inner surface of the throat and exit cone was machined to a 500-micron finish.

4 Oval exit cone—same as Test 1 except that the exit cone was oval by 0.040 in.

5 Cast nozzle—using a nozzle with "as-cast" finish, except in the throat. The cast exit cone was welded to the throat section and the weld extending into the gas-flow stream was removed.

6 Thrust ring—same at Test 1 except that the rocket was mounted from the thrust ring on the booster case instead of from the nozzle.

The data from Tests 5 and 6 are reproduced as Figs. 11 and 12, respectively, since these tests are perhaps the most interesting. In general, Test 1 resulted in a deviation angle of 0.05 deg; and Test 2, although resulting in a deviation angle of approximately 0.10 deg, did not differ sufficiently from Test 1 for any effect to be attributed to the more gradual throat radius. Test 3 also resulted in an angle of deviation of approximately 0.10 deg and was not significantly different from Test 1, so that it may be concluded that no effect on thrust alignment occurs within the range from 100 to 500-micron surface finish. In Test 4 a deviation, greater than that obtained in the preceding three tests, resulted. It is therefore concluded that present close tolerances on ovality in rocket exit cones are justified.

Test 5 (Fig. 11) resulted in quite high values for thrust misalignment. An angle of 0.4 deg, corresponding to a lateral force in excess of 80 lb, was obtained. The nozzle used in this test was cast in two pieces: the entrance section and throat, and the exit cone. The two parts were welded together and, although the projection of weld material in the exit cone was ground smooth, the contour in this section was not continuous. Also, there was some ovality in the exit cone as a result of the warpage during welding. Although the axis of the exit cone diverged from the axis of the test fixture by only 0.06 deg, the local discontinuities affected the gas flow enough to result in the measured deviation of 0.4 deg. Thus, it is concluded that the present practice of contour-machining the interior surface of the throat and exit cone should be maintained.

Test 6 (Fig. 12) resulted in an angular deviation of less than 0.1 deg and very small values for the X_0 and Z_0 components of the deviation. These values compare favorably with those obtained in Tests 1, 2, and 3; thus, it is concluded that effects of differential thermal expansion between the nozzle and the thrust ring are negligible.

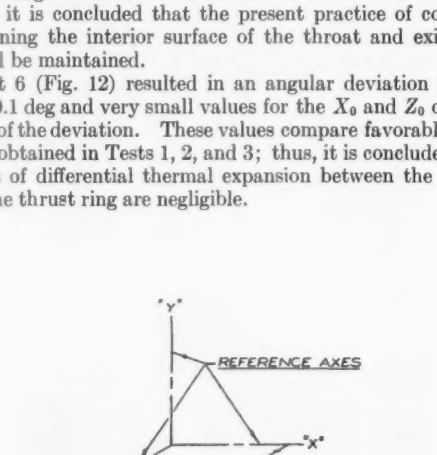


FIG. 10 COORDINATES USED TO DEFINE THRUST VECTOR POSITION

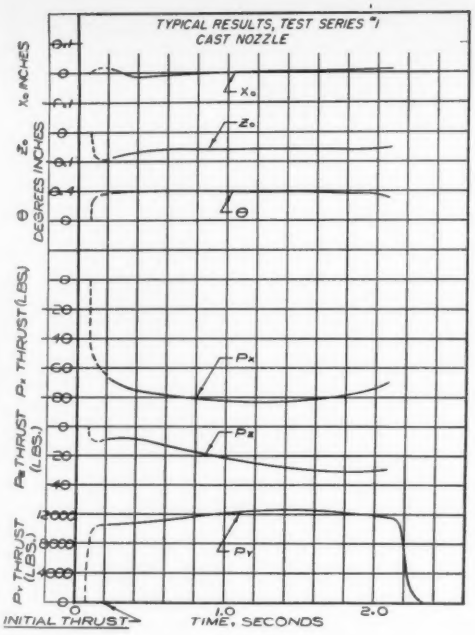


FIG. 11 TYPICAL RESULTS, TEST SERIES #1. CAST NOZZLE

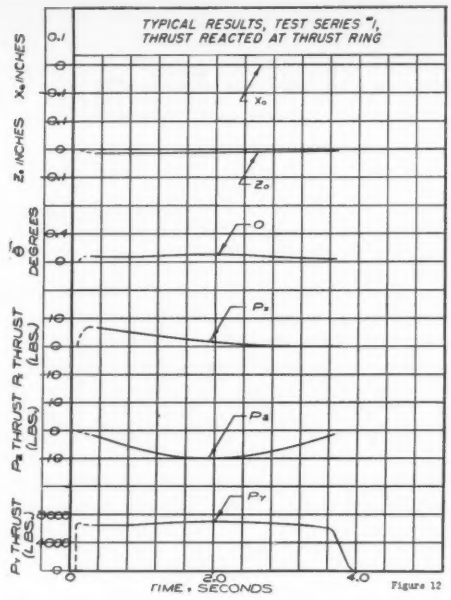


FIG. 12 TYPICAL RESULTS, TEST SERIES #1. THRUST REACTED AT THRUST RING

Series 2

Thrust-alignment firings were made with a regeneratively cooled acid-aniline motor. The propellant tanks were mounted behind the test-pit wall and the propellant lines were conducted through the wall and into the fixture near the lower end and thence upward to the motor. Because of the small deflection of the fixture underload, no flexible joints in the lines were necessary. Correction for the forces imposed by the propellant flow was accomplished by measuring these forces with water flowing in the lines under comparable conditions to propellant flow during firing. At worst, this correction amounted to approximately 2 lb.

The usual data on the position of the center line of the nozzle before and after firing were taken, and, in addition, extensometer data on the expansion of the liquid-motor cooling-jacket bellows were obtained. Attempts were made to correlate the measured position of the line of thrust with these data, but no correlation was found. Of course, the actual position of the geometric center line of the nozzle during firing is not necessarily represented by any of these data, since the extensometer readings measure only longitudinal expansion and since the measurement of the position of the nozzle center line after firing is undoubtedly affected by the cold-water flush used at the end of each firing to protect the motor from damage.

Data from the test in which maximum misalignment was obtained are included in Fig. 13. The symbols used are the same as those shown in Fig. 10, with the addition of the angle α, which is the angle made with the X-Y plane by the vertical plane in which the thrust vector lies.

The misalignment at the beginning of the test is 0.18 deg, which is comparable to the misalignment obtained with the solid propellant rockets described above. As the test progresses, however, the misalignment increases to a maximum of approximately 0.45 deg. This indicates that a change in the configuration of the motor occurs as the run progresses. The dimensions X_0 and Z_0 are quite small, generally less than 0.010 in. As in the test shown in Fig. 12, all of the tests resulted in change in θ during the course of the run to a greater or less degree, but the variation was random.

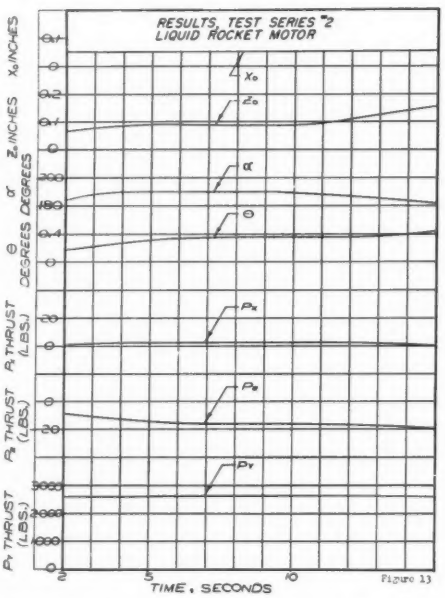


FIG. 13 RESULTS, TEST SERIES #2. LIQUID ROCKET MOTOR

The data obtained thus far in the test facility have proved of great value in resolving questions in regard to thrust alignment. As guided missiles progress through the development stage and into Service use, this facility will be available to obtain data which can be used to establish realistic requirements for the manufacture of rocket engines so that the most economical fabrication consistent with missile requirements can be achieved.

Minimal Rocket Trajectories

D. F. LAWDEN[1]

College of Technology, Birmingham, England

The problem of the navigation of a rocket vehicle between two fixed terminals in space, in such a manner as to minimize the fuel expenditure, is analyzed in a general fashion, account being taken of a gravitational field, variable in space and time. The motion is assumed to take place in vacuo. The transit time may or may not be regarded as specified. The solution trajectory is shown to comprise a number of null-thrust arcs, separated by junctions at which impulsive thrusts are applied. The conditions to be satisfied at these junctions are calculated. The theory is applied to the special case of the optimal transfer of a rocket between two coplanar elliptical orbits about a center of attraction.

1 Introduction

THE object of this paper is to solve in general terms the problem of the transfer of a rocket between two terminals, at which its velocity is to be as specified, the trajectory of transfer and the programming of fuel expenditure to be such that the over-all fuel consumption is minimized. No restrictions will be placed upon the gravitational field to which the vehicle is supposed subject, but the motion will be supposed to take place in vacuo, so that atmospheric resistance forces are absent. The solution obtained is therefore applicable to problems concerning the navigation of a space ship between two points of the solar system, when it is necessary to take account of the fields of various gravitating bodies. Particular problems of this type have already been studied by the author (1),[2] (2), (3). Problems of a similar type, in which air resistance has been taken into account but the trajectory has been assumed rectilinear and gravity uniform, have been discussed by Tsien and Evans (4) and by Hibbs (5).

A feature common to the solutions to all problems of this nature so far studied, is the occurrence of impulsive thrusts. Thus Tsien and Evans require that for optimum performance of a vertically ascending rocket, an initial impulsive thrust shall be applied to bring the velocity up to a certain critical value, and this they propose shall be generated by a booster which falls away as the rocket proper leaves its launching tower. Hibbs shows that if a rocket craft is to achieve maximum range in horizontal flight over a flat earth, its launching velocity must take a certain critical value, this being achieved by means of an initial impulsive thrust. In the simple case of the optimum transfer of a rocket between two points of a uniform gravitational field, all other resistive forces being neglected, the author has proved (6) that it is necessary to apply impulsive thrusts at each terminal, the rocket being in a state of free fall during the period of transit. That such impulses must always be present in the solution to the general problem, will be found demonstrated in (3). The existence of impulses in the solution will clearly complicate the mathematical analysis of the problem. Another, and greater difficulty, is that any trajectory of null-thrust is clearly a minimal trajectory, and yet it is found that the orthodox Euler characteristic equations for this problem are indeterminate for such a solution. For this reason, we shall find it convenient to solve an allied mathematical problem in which this latter anomaly does not arise, but which can be made to approach our actual problem as a limit.

Received March 6, 1953.

[1] Senior Lecturer in Mathematics. Fellow of the British Interplanetary Society.

[2] Numbers in parentheses refer to the References on page 382.

2 The General Theory

We shall first consider the problem of the optimum transfer of a rocket between two terminals lying in a plane, it being assumed that all gravitational forces and the jet thrust are parallel to this plane and that the rocket trajectory is contained in the plane. This restriction to two dimensions is not essential, but serves to reduce somewhat the number of terms occurring in our expressions. Our final results will be generalizable in an obvious manner to the three-dimensional case.

Let Ox, Oy be fixed rectangular Cartesian axes in the plane of motion, and let (l, m) be the direction cosines of the direction of the jet thrust at the instant t when the rocket is at the point (x, y). If c is the exhaust velocity and M is the rocket mass, we may write the equations of motion in the form

$$\ddot{x} + f = -\frac{lc}{M}\frac{dM}{dt} \qquad \ddot{y} + g = -\frac{mc}{M}\frac{dM}{dt} \quad \ldots\ldots [1]$$

where dots denote differentiations with respect to the time variable and $(-f, -g)$ are the components of the resultant force per unit mass acting on the rocket due to gravity. It is not assumed that the direction of motion and of thrust are necessarily coincident, nor is it assumed that the gravitational field is steady and therefore independent of t. We shall take f and g to depend explicitly upon the variables t, x, y only.

Since $l^2 + m^2 = 1$, it follows from Equations [1] that

$$-\frac{c}{M}\frac{dM}{dt} = \{(\ddot{x} + f)^2 + (\ddot{y} + g)^2\}^{1/2} \quad \ldots\ldots [2]$$

whence, by integration over a time interval (t_0, t_1), we obtain

$$c \log \frac{M_0}{M_1} = \int_{t_0}^{t_1} \{(\ddot{x} + f)^2 + (\ddot{y} + g)^2\}^{1/2}\, dt \quad \ldots\ldots [3]$$

M_0, M_1 being the mass of the rocket at the commencement and end of the interval, respectively.

If the rocket is to be transferred from a point (x_0, y_0), at which its velocity is (u_0, v_0), to a point (x_1, y_1), at which its velocity of arrival is to be (u_1, v_1), the navigational problem is to select functions $x(t)$ and $y(t)$, determining the trajectory of transit, subject to the boundary conditions

$$\begin{array}{llll} x(t_0) = x_0 & \dot{x}(t_0) = u_0 & x(t_1) = x_1 & \dot{x}(t_1) = u_1 \\ y(t_0) = y_0 & \dot{y}(t_0) = v_0 & y(t_1) = y_1 & \dot{y}(t_1) = v_1 \end{array} \quad \ldots\ldots [4]$$

From the field of such functions, we shall show how to determine those which minimize the mass ratio M_0/M_1, i.e., the integral in Equation [3] is to be minimized relative to variation of the functions $x(t)$, $y(t)$ and of the interval of transit (t_0, t_1).

Before proceeding to an analysis of this general problem, we shall consider the very special case when $f = g = 0$, our object being to illustrate the difficulties which inevitably arise in the general case. We have then to minimize the integral

$$\int_{t_0}^{t_1} (\ddot{x}^2 + \ddot{y}^2)^{1/2}\, dt \quad \ldots\ldots [5]$$

under the boundary conditions [4]. Assuming a specified transit time, the Euler characteristic equations for the problem are

$$\frac{d^2}{dt^2}\left[\frac{\ddot{x}}{(\ddot{x}^2+\ddot{y}^2)^{1/2}}\right]=\frac{d^2}{dt^2}\left[\frac{\ddot{y}}{(\ddot{x}^2+\ddot{y}^2)^{1/2}}\right] \quad \ldots\ldots [6]$$

or $\dot{l}=\dot{m}=0$ (l, m) being the direction cosines of the thrust. Integration provides us with the general solutions

$$l=A+Bt \qquad m=C+Dt \quad \ldots\ldots [7]$$

but in view of the identity $l^2+m^2=1$, we must take $B=D=0$ and A and C such that $A^2+C^2=1$, i.e., the direction of thrust must remain constant during the maneuver. Only one independent arbitrary constant therefore remains and it is not possible to satisfy all eight of the boundary conditions [4] in general. This inadequacy of the characteristic equations extends to the general case (5), and it therefore proves to be necessary to minimize the integral

$$I=\int_{t_0}^{t_1}\{(\ddot{x}+f)^2+(\ddot{y}+g)^2+\epsilon^2\}^{1/2}\,dt \quad \ldots\ldots [8]$$

finally letting $\epsilon \to 0$. The characteristic equations corresponding to this latter integral are nonsingular and their general solution contains its full complement of arbitrary constants.

If we integrate Equations [1] over the short interval i of application of an impulsive thrust, we obtain

$$lc \log r=|\dot{x}|_i \qquad mc \log r=|\dot{y}|_i \quad \ldots\ldots [9]$$

where $|\dot{x}|_i$, $|\dot{y}|_i$ denote the increments in the two components of velocity due to the impulse, and r is the mass ratio for the maneuver. We are justified in neglecting the integrals $\int_i f\,dt$, $\int_i g\,dt$ since f and g remain finite during i. The direction (l, m) has been supposed not to vary during the thrust.

Squaring and adding Equations [9] we find that

$$c \log r=\sqrt{|\dot{x}|_i^2+|\dot{y}|_i^2} \quad \ldots\ldots [10]$$

This implies that the contribution of such an impulse to the integral [3] is equal to the magnitude of the vector velocity increment it causes. The corresponding contribution to the integral [8] will be the same, since ϵ also remains finite like f and g and hence may be neglected by comparison with $\ddot{x}$ and $\ddot{y}$.

We now make t and hence x and y depend upon a parameter λ of no physical significance and such that, as λ increases monotonically from α to β, t increases from t_0 to t_1. Denoting differentiations with respect to λ by primes, we have

$$\dot{x}=\frac{x'}{t'} \quad \dot{y}=\frac{y'}{t'} \quad \ddot{x}=\frac{t'x''-t''x'}{t'^3} \quad \ddot{y}=\frac{t'y''-t''y'}{t'^3} \quad \ldots [11]$$

and hence the integral I is of the form

$$I=\int_{t_0}^{t_1} h(t, x, y, \dot{x}, \dot{y})\,dt=\int_\alpha^\beta H(t, x, y, t', x', y', t'', x'', y'')\,d\lambda \quad \ldots [12]$$

where

$$H(t, x, y, t', x', y', t'', x'', y'')=t'h(t, x, y, \dot{x}, \dot{y}) \quad \ldots\ldots [13]$$

Suppose $x=x(\lambda)$, $y=y(\lambda)$, $t=t(\lambda)$, $(\alpha \leqslant \lambda \leqslant \beta)$ specifies a rocket trajectory satisfying the boundary conditions [4] and corresponding to a minimum expenditure of fuel. We shall assume initially that one impulsive thrust only is applied and this when $\lambda=\gamma$. Having found the conditions to be satisfied on either side of such a thrust, we will then remove this restriction. Let V be the magnitude of the velocity increment due to this impulse. Equation [12] can then be written

$$I=\int_\alpha^{\gamma-0} H\,d\lambda+\int_{\gamma+0}^\beta H\,d\lambda+V \quad \ldots\ldots [14]$$

If now $x=x(\lambda)+\delta x(\lambda)$, $y=y(\lambda)+\delta y(\lambda)$, $t=t(\lambda)+\delta t(\lambda)$, $(\alpha \leqslant \lambda \leqslant \beta)$ defines a neighboring rocket trajectory, satisfying the boundary conditions [4] at the instants $t_0+\delta t_0$, $t_1+\delta t_1$, respectively, the impulse being varied so that the velocity increment caused by it is altered by δV, then from Equation [14]

$$\delta I=\int_\alpha^{\gamma-0}+\int_{\gamma+0}^\beta\left[\frac{\partial H}{\partial t}\delta t+\frac{\partial H}{\partial x}\delta x+\frac{\partial H}{\partial y}\delta y+\frac{\partial H}{\partial t'}\delta t'+\frac{\partial H}{\partial x'}\delta x'+\frac{\partial H}{\partial y'}\delta y'+\frac{\partial H}{\partial t''}\delta t''+\frac{\partial H}{\partial x''}\delta x''+\frac{\partial H}{\partial y''}\delta y''\right]d\lambda+\delta V \quad [15]$$

The following notation will now be adopted. Variables which are to be given their values at $\lambda=\alpha$ will have a subscript 0 attached, and those which are to be calculated at $\lambda=\beta$ will have a subscript 1 attached. A variable which is to take its value immediately prior to the impulse, will be modified by the subscript $-$, whereas, if it is to be calculated immediately after the impulse, a subscript $+$ will be shown.

Then

$$\delta V=\delta\{(\dot{x}_+-\dot{x}_-)^2+(\dot{y}_+-\dot{y}_-)^2\}^{1/2}=\frac{(\dot{x}_+-\dot{x}_-)(\delta\dot{x}_+-\delta\dot{x}_-)+(\dot{y}_+-\dot{y}_-)(\delta\dot{y}_+-\delta\dot{y}_-)}{\{(\dot{x}_+-\dot{x}_-)^2+(\dot{y}_+-\dot{y}_-)^2\}^{1/2}} \quad \ldots [16]$$

But

$$\dot{x}=x'/t'$$

$$\delta\dot{x}=\frac{t'\delta x'-x'\delta t'}{t'^2} \quad \ldots\ldots [17]$$

with similar equations for $\dot{y}$ and $\delta\dot{y}$. Substituting from Equations [17] into Equation [16] we obtain

$$\delta V=\frac{(x_+'t_-'-x_-'t_+')\{t_-'^2(t_+'\delta x_+'-x_+'\delta t_+')-t_+'^2(t_-'\delta x_-'-x_-'\delta t_-')\}+(y_+'t_-'-y_-'t_+')\{t_-'^2(t_+'\delta y_+'-y_+'\delta t_+')-t_+'^2(t_-'\delta y_-'-y_-'\delta t_-')\}}{t_+'^2t_-'^2\{(x_+'t_-'-x_-'t_+')^2+(y_+'t_-'-y_-'t_+')^2\}^{1/2}} \quad [18]$$

This expression for δV may now be set in Equation [15]. Integrations by parts simplify the various integrals occurring in this equation thus:

$$\int\frac{\partial H}{\partial t'}\delta t'\,d\lambda=\frac{\partial H}{\partial t'}\delta t-\int\frac{d}{d\lambda}\left(\frac{\partial H}{\partial t'}\right)\delta t\,d\lambda$$

$$\int\frac{\partial H}{\partial t''}\delta t''\,d\lambda=\frac{\partial H}{\partial t''}\delta t'-\int\frac{d}{d\lambda}\left(\frac{\partial H}{\partial t''}\right)\delta t'\,d\lambda=\frac{\partial H}{\partial t''}\delta t'-\frac{d}{d\lambda}\left(\frac{\partial H}{\partial t''}\right)\delta t+\int\frac{d^2}{d\lambda^2}\left(\frac{\partial H}{\partial t''}\right)\delta t\,d\lambda \quad [19]$$

etc. It now follows that Equation [15] is of the form

$$\begin{aligned}\delta I=\;&-A_0\delta t_0+A_-\delta t_-+B_-\delta x_-+C_-\delta y_-+A_+\delta t_+\\&-B_+\delta x_+-C_+\delta y_++A_1\delta t_1-G_0\delta t_0'-J_0\delta x_0'\\&-K_0\delta y_0'+L\delta t_-'+M\delta x_-'+N\delta y_-'-P\delta t_+'\\&-Q\delta x_+'-R\delta y_+'+G_1\delta t_1'+J_1\delta x_1'+K_1\delta y_1'\\&+\int_\alpha^{\gamma-0}+\int_{\gamma+0}^\beta(T\delta t+X\delta x+Y\delta y)\,d\lambda \quad \ldots\ldots [20]\end{aligned}$$

since $\delta x_0=\delta y_0=\delta x_1=\delta y_1=0$ by application of the boundary conditions [4]. The expressions A, B, etc., are listed as follows:

$$A = \frac{\partial H}{\partial t'} - \frac{d}{d\lambda}\left(\frac{\partial H}{\partial t''}\right) \qquad B = \frac{\partial H}{\partial x'} - \frac{d}{d\lambda}\left(\frac{\partial H}{\partial x''}\right)$$

$$C = \frac{\partial H}{\partial y'} - \frac{d}{d\lambda}\left(\frac{\partial H}{\partial y''}\right)$$

$$G = \frac{\partial H}{\partial t''} \qquad J = \frac{\partial H}{\partial x''} \qquad K = \frac{\partial H}{\partial y''}$$

$$L = \left(\frac{\partial H}{\partial t''}\right)_- + \frac{x_-'(x_+'t_-' - x_-t_+') + y_-'(y_+'t_-' - y_-'t_+')}{t_-'^2\left\{(x_+'t_-' - x_-'t_+')^2 + (y_+'t_-' - y_-'t_+')^2\right\}^{1/2}}$$

$$M = \left(\frac{\partial H}{\partial x''}\right)_- - \frac{x_+'t_-' - x_-'t_+'}{t_-'\left\{(x_+'t_-' - x_-'t_+')^2 + (y_+'t_-' - y_-'t_+')^2\right\}^{1/2}}$$

$$N = \left(\frac{\partial H}{\partial y''}\right)_- - \frac{y_+'t_-' - y_-'t_+'}{t_-'\left\{(x_+'t_-' - x_-'t_+')^2 + (y_+'t_-' - y_-'t_+')^2\right\}^{1/2}}$$

$$P = \left(\frac{\partial H}{\partial t''}\right)_+ + \frac{x_+'(x_+'t_-' - x_-'t_+') + y_+'(y_+'t_-' - y_-'t_+')}{t_+'^2\left\{(x_+'t_-' - x_-'t_+')^2 + (y_+'t_-' - y_-'t_+')^2\right\}^{1/2}}$$

$$Q = \left(\frac{\partial H}{\partial x''}\right)_+ - \frac{x_+'t_-' - x_-'t_+'}{t_+'\left\{(x_+'t_-' - x_-'t_+')^2 + (y_+'t_-' - y_-'t_+')^2\right\}^{1/2}}$$

$$R = \left(\frac{\partial H}{\partial y''}\right)_+ - \frac{y_+'t_-' - y_-'t_+'}{t_+'\left\{(x_+'t_-' - x_-'t_+')^2 + (y_+'t_-' - y_-'t_+')^2\right\}^{1/2}}$$

$$T = \frac{d^2}{d\lambda^2}\left(\frac{\partial H}{\partial t''}\right) - \frac{d}{d\lambda}\left(\frac{\partial H}{\partial t'}\right) + \frac{\partial H}{\partial t}$$

$$X = \frac{d^2}{d\lambda^2}\left(\frac{\partial H}{\partial x''}\right) - \frac{d}{d\lambda}\left(\frac{\partial H}{\partial x'}\right) + \frac{\partial H}{\partial x}$$

$$Y = \frac{d^2}{d\lambda^2}\left(\frac{\partial H}{\partial y''}\right) - \frac{d}{d\lambda}\left(\frac{\partial H}{\partial y'}\right) + \frac{\partial H}{\partial y} \quad \ldots\ldots [21]$$

Since $\dot{x} = x'/t'$ and $\dot{y} = y'/t'$ are prescribed by the boundary conditions at the terminals, the quantities

$$\delta\dot{x} = \frac{t'\delta x' - x'\delta t'}{t'^2} \qquad \delta\dot{y} = \frac{t'\delta y' - y'\delta t'}{t'^2}$$

are zero at $\lambda = \alpha$ and $\lambda = \beta$. Hence

$$\delta x' = x'\delta t'/t' \qquad \delta y' = y'\delta t'/t' \quad \ldots\ldots [22]$$

at these instants. At $\lambda = \alpha$ or $\lambda = \beta$ therefore

$$G\delta t' + J\delta x' + K\delta y' = \frac{\delta t'}{t'}\left(\frac{\delta H}{\partial t''}t' + \frac{\partial H}{\partial x''}x' + \frac{\partial H}{\partial y''}y'\right) \ldots [23]$$

Equations [11] and [13] yield

$$\frac{\partial H}{\partial t''} = t'\left(\frac{\partial h}{\partial \ddot{x}}\frac{\partial \ddot{x}}{\partial t''} + \frac{\partial h}{\partial \ddot{y}}\frac{\partial \ddot{y}}{\partial t''}\right) = -\frac{x'}{t'^2}\frac{\partial h}{\partial \ddot{x}} - \frac{y'}{t'^2}\frac{\partial h}{\partial \ddot{y}}$$
$$= -\frac{1}{t'}\left(\dot{x}\frac{\partial h}{\partial \ddot{x}} + \dot{y}\frac{\partial h}{\partial \ddot{y}}\right) \ldots [24]$$

$$\frac{\partial H}{\partial x''} = t'\frac{\partial h}{\partial \ddot{x}}\frac{\partial \ddot{x}}{\partial x''} = \frac{1}{t'}\frac{\partial h}{\partial \ddot{x}} \quad \ldots\ldots [25]$$

$$\frac{\partial H}{\partial y''} = t'\frac{\partial h}{\partial \ddot{y}}\frac{\partial \ddot{y}}{\partial y''} = \frac{1}{t'}\frac{\partial h}{\partial \ddot{y}} \quad \ldots\ldots [26]$$

Substitution of these results in Equation [23] now proves that

$$G\delta t' + J\delta x' + K\delta y' = 0$$

at either terminal. Equation [20] accordingly reduces to

$$\delta I = -A_0\delta t_0 + A_-\delta t_- + B_-\delta x_- + C_-\delta y_- - A_+\delta t_+ - B_+\delta x_+ - C_+\delta y_+ + A_1\delta t_1 + L\delta t_-' + M\delta x_-' + N\delta y_-' - P\delta t_+' - Q\delta x_+' - R\delta y_+' + \int_\alpha^{\gamma-0} + \int_{\gamma+0}^{\beta}(T\delta t + X\delta x + Y\delta y)\,d\lambda \quad \ldots\ldots [27]$$

But since δt, δx, δy are continuous over the impulse, $\delta t_- = \delta t_+$, $\delta x_- = \delta x_+$, $\delta y_- = \delta y_+$. Apart from these relationships, all the quantities δt_0, δt_-, δx_-, etc., are arbitrary, and since the integral I is being varied about its minimum value, the first variation δI is zero for all arbitrary small variations of the trajectory, and this can only be so if

$$A_0 = A_+ - A_- = B_+ - B_- = C_+ - C_- = A_1 = 0$$
$$L = M = N = P = Q = R = 0$$
$$X \equiv Y \equiv T \equiv 0 \quad \ldots\ldots [28]$$

If the instants of departure and arrival are prescribed, $\delta t_0 = \delta t_1 = 0$ and we can dispense with the conditions $A_0 = A_1 = 0$. If the time of transit only is specified, $\delta t_0 = \delta t_1$ but are otherwise arbitrary. In this case, $A_0 = A_1$ but are not necessarily zero.

The last three of Equations [28] are differential equations for the characteristic curves and it is well known (7) that only two of these are independent. Putting $\lambda = t$ in the equations $X = Y = 0$, we obtain two independent equations for these curves in the form

$$\frac{d^2}{dt^2}\left(\frac{\partial h}{\partial \ddot{x}}\right) + \frac{\partial h}{\partial x} = 0$$
$$\frac{d^2}{dt^2}\left(\frac{\partial h}{\partial \ddot{y}}\right) + \frac{\partial h}{\partial y} = 0 \quad \ldots\ldots [29]$$

Referring again to Equations [11] and [13] we obtain

$$\frac{\partial H}{\partial t'} = h + \left(\frac{\partial h}{\partial \ddot{x}}\frac{\partial \ddot{x}}{\partial t'} + \frac{\partial h}{\partial \ddot{y}}\frac{\partial \ddot{y}}{\partial t'}\right)t'$$
$$= h + \left(\frac{3t''x' - 2t'x''}{t'^3}\right)\frac{\partial h}{\partial \ddot{x}} + \left(\frac{3t''y' - 2t'y''}{t'^3}\right)\frac{\partial h}{\partial \ddot{y}} \quad \ldots\ldots [30]$$

$$\frac{\partial H}{\partial x'} = t'\frac{\partial h}{\partial \ddot{x}}\frac{\partial \ddot{x}}{\partial x'} = -\frac{t''}{t'^2}\frac{\partial h}{\partial \ddot{x}} \quad \ldots\ldots [31]$$

$$\frac{\partial H}{\partial y'} = t'\frac{\partial h}{\partial \ddot{y}}\frac{\partial \ddot{y}}{\partial y'} = -\frac{t''}{t'^2}\frac{\partial h}{\partial \ddot{y}} \quad \ldots\ldots [32]$$

Equations [24]–[26] together with these latter results now imply that

$$A = h - \ddot{x}\frac{\partial h}{\partial \ddot{x}} - \ddot{y}\frac{\partial h}{\partial \ddot{y}} + \dot{x}\frac{d}{dt}\left(\frac{\partial h}{\partial \ddot{x}}\right) + \dot{y}\frac{d}{dt}\left(\frac{\partial h}{\partial \ddot{y}}\right) \ldots [33]$$

$$B = -\frac{d}{dt}\left(\frac{\partial h}{\partial \ddot{x}}\right) \quad \ldots\ldots [34]$$

$$C = -\frac{d}{dt}\left(\frac{\partial h}{\partial \ddot{y}}\right) \quad \ldots\ldots [35]$$

Conditions [28] require that these three expressions shall be continuous over the impulse. In addition, if the instants of departure and arrival are not prescribed, the expression [33] must vanish at both terminals.

Consider now the conditions $M = Q = 0$. Employing Equation [25], we find that these are equivalent to

$$\left(\frac{\partial h}{\partial \ddot{x}}\right)_- = \frac{\dot{x}_+ - \dot{x}_-}{\left\{(\dot{x}_+ - \dot{x}_-)^2 + (\dot{y}_+ - \dot{y}_-)^2\right\}^{1/2}} = \left(\frac{\partial h}{\partial \ddot{x}}\right)_+ \ldots [36]$$

Similarly, $N = R = 0$ yield the conditions

$$\left(\frac{\partial h}{\partial \ddot{y}}\right)_- = \frac{\dot{y}_+ - \dot{y}_-}{\left\{(\dot{x}_+ - \dot{x}_-)^2 + (\dot{y}_+ - \dot{y}_-)^2\right\}^{1/2}} = \left(\frac{\partial h}{\partial \ddot{y}}\right)_+ \ldots [37]$$

The identity

$$t'\frac{\partial H}{\partial t''} + x'\frac{\partial H}{\partial x''} + y'\frac{\partial H}{\partial y''} = 0 \quad \ldots\ldots [38]$$

follows from Equations [24]–[26], and hence the conditions $L = P = 0$ are linear combinations of the conditions expressed by Equations [36] and [37].

Since $h = \left\{(\ddot{x} + f)^2 + (\ddot{y} + g)^2 + \epsilon^2\right\}^{1/2}$, Equations [36] and [37] may be written

$$\frac{\ddot{x}+f}{\{(\ddot{x}+f)^2+(\ddot{y}+g)^2+\epsilon^2\}^{1/2}} = \frac{\dot{x}_+ - \dot{x}_-}{\{(\dot{x}_+ - \dot{x}_-)^2+(\dot{y}_+ - \dot{y}_-)^2\}^{1/2}}$$

$$\frac{\ddot{y}+g}{\{(\ddot{x}+f)^2+(\ddot{y}+g)^2+\epsilon^2\}^{1/2}} = \frac{\dot{y}_+ - \dot{y}_-}{\{(\dot{x}_+ - \dot{x}_-)^2+(\dot{y}_+ - \dot{y}_-)^2\}^{1/2}} \quad \ldots [39]$$

both equations to be satisfied immediately prior and immediately subsequent to the impulse. It is required therefore that both $(\ddot{x}+f)$ and $(\ddot{y}+g)$ shall be infinite in the ratio $(\dot{x}_+ - \dot{x}_-) : (\dot{y}_+ - \dot{y}_-)$ on both sides of the impulse. Further, since the square roots appearing in Equations [3] and [8] are to be taken positively, it follows that all square roots which take their origin from these are to be taken with the same sign. In particular, those roots appearing in the conditions [39] are positive and hence $\ddot{x}+f$ and $\dot{x}_+ - \dot{x}_-$ are of like sign, as are the pair $\ddot{y}+g$ and $\dot{y}_+ - \dot{y}_-$. This being understood, we have the conditions

$$\frac{\ddot{x}+f}{\ddot{y}+g} = \frac{\dot{x}_+ - \dot{x}_-}{\dot{y}_+ - \dot{y}_-} \quad \ddot{x}+f = \ddot{y}+g = \infty \quad \ldots [40]$$

The physical significance of these conditions is clear after reference to Equations [1], viz., the direction of rocket thrust immediately before and immediately after an impulse must coincide with the direction of the impulsive thrust, and the magnitude of the thrust must tend to infinity as the instant of application of the impulse is approached from either side.

We now proceed to calculate the quantities A, B, C as given by Equations [33]–[35]. Substituting for h we find that

$$A = \frac{(\ddot{x}+f)f+(\ddot{y}+g)g+\epsilon^2}{\{(\ddot{x}+f)^2+(\ddot{y}+g)^2+\epsilon^2\}^{1/2}} + \dot{x}\frac{d}{dt}\left[\frac{\ddot{x}+f}{\{(\ddot{x}+f)^2+(\ddot{y}+g)^2+\epsilon^2\}^{1/2}}\right] + \dot{y}\frac{d}{dt}\left[\frac{\ddot{y}+g}{\{(\ddot{x}+f)^2+(\ddot{y}+g)^2+\epsilon^2\}^{1/2}}\right] \quad \ldots [41]$$

$$B = -\frac{d}{dt}\left[\frac{\ddot{x}+f}{\{(\ddot{x}+f)^2+(\ddot{y}+g)^2+\epsilon^2\}^{1/2}}\right] \quad \ldots [42]$$

$$C = -\frac{d}{dt}\left[\frac{\ddot{y}+g}{\{(\ddot{x}+f)^2+(\ddot{y}+g)^2+\epsilon^2\}^{1/2}}\right] \quad \ldots [43]$$

We have now found the conditions to be satisfied in the neighborhood of any impulse which is applied at an instant lying between t_0 and t_1. If an impulse is to be applied at either of these end points, a similar analysis shows that the direction of the rocket thrust must be continuous as we pass from the impulsive régime into the finite thrust régime, and that the quantities $\ddot{x}+f$ and $\ddot{y}+g$ must tend to infinity as the impulse is approached. The signs of these quantities are also limited in the neighborhood of the impulse as explained above. No other conditions have to be satisfied at such points.

Turning now to the characteristic equations determining the trajectory in the intervals between impulses, we substitute for h in Equations [29] and so obtain the following equations

$$\frac{d^2}{dt^2}\left[\frac{\ddot{x}+f}{\{(\ddot{x}+f)^2+(\ddot{y}+g)^2+\epsilon^2\}^{1/2}}\right] + \frac{(\ddot{x}+f)\dfrac{\partial f}{\partial x}+(\ddot{y}+g)\dfrac{\partial g}{\partial x}}{\{(\ddot{x}+f)^2+(\ddot{y}+g)^2+\epsilon^2\}^{1/2}} = 0$$

$$\frac{d^2}{dt^2}\left[\frac{\ddot{y}+g}{\{(\ddot{x}+f)^2+(\ddot{y}+g)^2+\epsilon^2\}^{1/2}}\right] + \frac{(\ddot{x}+f)\dfrac{\partial f}{\partial y}+(\ddot{y}+g)\dfrac{\partial g}{\partial y}}{\{(\ddot{x}+f)^2+(\ddot{y}+g)^2+\epsilon^2\}^{1/2}} = 0 \quad \ldots [44]$$

We will first look for solutions of the form

$$x = X + \epsilon\xi \qquad y = Y + \epsilon\eta$$

where X and Y are any functions satisfying the equations

$$\ddot{x}+f = \ddot{y}+g = 0 \quad \ldots [46]$$

i.e., we shall determine solutions in the neighborhood of the trajectories of null-thrust and which will tend to the latter as $\epsilon \to 0$.

Substituting for x and y from Equations [45] we find that

$$\begin{aligned}\ddot{x}+f &= \ddot{X}+\epsilon\ddot{\xi}+f(t, X+\epsilon\xi, Y+\epsilon\eta)\\ &= \ddot{X}+f(t, X, Y)+\epsilon\left(\ddot{\xi}+\frac{\partial f}{\partial X}\xi+\frac{\partial f}{\partial Y}\eta\right)+0(\epsilon^2)\\ &= \epsilon p+0(\epsilon^2) \quad \ldots [47]\end{aligned}$$

where

$$p = \ddot{\xi}+\frac{\partial f}{\partial X}\xi+\frac{\partial f}{\partial Y}\eta \quad \ldots [48]$$

Similarly

$$\ddot{y}+g = \epsilon q+0(\epsilon^2) \quad \ldots [49]$$

where

$$q = \ddot{\eta}+\frac{\partial g}{\partial X}\xi+\frac{\partial g}{\partial Y}\eta \quad \ldots [50]$$

Substituting for $\ddot{x}+f$ and $\ddot{y}+g$ from these latter equations in Equations [44], we find that these may be written

$$\begin{aligned}\ddot{u}+u\frac{\partial f}{\partial X}+v\frac{\partial g}{\partial X} &= 0\\ \ddot{v}+u\frac{\partial f}{\partial Y}+v\frac{\partial g}{\partial Y} &= 0 \quad \ldots [51]\end{aligned}$$

where

$$u = p/(p^2+q^2+1)^{1/2} \qquad v = q/(p^2+q^2+1)^{1/2} \quad \ldots [52]$$

and terms $0(\epsilon)$ have been neglected.

Solving for the functions p and q in terms of u and v, we obtain

$$p = u/(1-u^2-v^2)^{1/2} \qquad q = v/(1-u^2-v^2)^{1/2} \quad \ldots [53]$$

The functions p and q are completely determined by these equations, apart from terms $0(\epsilon)$ which were neglected in Equations [51]. Equations [48] and [50] then specify ξ, η to the same order of accuracy, and Equations [45] then fix (x, y) to $0(\epsilon)$.

We shall assume the functions f and g to be differentiable any number of times with respect to all their variables. X and Y are then differentiable functions of t to any order, as are the functions $\partial f/\partial X$, etc. Equations [51] therefore define u and v as differentiable functions of t and, in particular, these functions will be bounded for finite values of t. Referring to Equations [53], we see that in this case p and q are differentiable functions of t, but will both tend to infinity as (u^2+v^2) approaches unity. Except in the neighborhood of such an instant, ξ and η, as given by Equations [48] and [50], will therefore be differentiable and bounded. As we approach an instant where $u^2+v^2 = 1$, however, p and q tend to infinity and Equations [48] and [50] reveal that $\ddot{\xi} \to \infty$ and $\ddot{\eta} \to \infty$, though ξ and η will remain finite. In view of the boundedness of these latter quantities, the quantities $\epsilon\xi$, $\epsilon\eta$, will remain small with ϵ and the approximations made in order to derive Equations [51] from Equations [44] remain valid in such a neighborhood. If therefore we integrate Equations [47]–[53] for the functions ξ and η and substitute in Equations [45], the resulting solution for x and y will be valid even in the vicinity of an instant when $u^2+v^2 = 1$. As we approach such an instant, however, although x and y will remain finite, $\ddot{x} \to \infty$ and $\ddot{y} \to \infty$ with $\ddot{\xi}$ and $\ddot{\eta}$. Such an instant is

therefore an appropriate one at which to apply an impulsive thrust.

Suppose then that integral [8] has been minimized, relative to the boundary conditions [4], by trajectory arcs specified by equations of the form given at [45], these arcs being connected at junctions at which $u^2 + v^2 = 1$ and at which impulsive thrusts are applied. Suppose that the conditions, which we have found must be satisfied at end points and at such junctions, are indeed satisfied. We now let $\epsilon \to 0$ to obtain the corresponding solution to our actual problem. Since ξ, η are bounded everywhere, Equations [45] show that the trajectory arcs tend to arcs of null-thrust. In the limit therefore, we arrive at a number of null-thrust arcs, separated by junctions at which impulsive thrusts are applied. It remains to calculate the limiting forms of the conditions to be satisfied at these junctions and the two end points.

Suppose that $x = X_1, y = Y_1$ and $x = X_2, y = Y_2$ represent two parts of the minimal rocket trajectory separated by an impulsive thrust when ϵ is small, but nonzero. Associated with the first arc there will be a solution of Equations [51] $u = u_1$, $v = v_1$, and with the second arc a solution $u = u_2$, $v = v_2$. At the impulse $u_1^2 + v_1^2 = u_2^2 + v_2^2 = 1$. In addition, Equations [40] require that on either side of the impulse

$$\frac{\dot{x}_+ - \dot{x}_-}{\dot{y}_+ - \dot{y}_-} = \frac{\ddot{x} + f}{\ddot{y} + g} = \frac{\epsilon p + 0(\epsilon^2)}{\epsilon q + 0(\epsilon^2)} = \frac{p}{q} + 0(\epsilon)$$

As $\epsilon \to 0$, this condition approaches the limiting form

$$\frac{\dot{x}_+ - \dot{x}_-}{\dot{y}_+ - \dot{y}_-} = \frac{p}{q} = \frac{u}{v} \quad \ldots\ldots [54]$$

Physically, these conditions mean that on either side of an impulse, (u, v) must represent the direction cosines of the direction of thrust.

Equations [53] show that if p and q are to be real, at all points of the trajectory, it is necessary that $u^2 + v^2 \leqslant 1$.

For the type of trajectory specified by Equations [45], the quantities A, B, C, as determined by Equations [41]–[43], may be expressed, by using Equations [47] and [49] in the form

$$\begin{aligned} A &= uf + vg + \dot{X}\dot{u} + \dot{Y}\dot{v} + 0(\epsilon) \\ B &= -\dot{u} + 0(\epsilon) \\ C &= -\dot{v} + 0(\epsilon) \end{aligned}$$

Letting $\epsilon \to 0$, these approach the limiting forms

$$A = uf + vg + \dot{X}\dot{u} + \dot{Y}\dot{v}, \quad B = -\dot{u}, \quad C = -\dot{v} \ldots [55]$$

It is therefore a requirement that these three quantities shall be continuous across an impulse when $\epsilon = 0$. Since (u, v) tend to the direction cosines of the direction of thrust as we approach a junction from either side, these quantities are continuous across such a junction. The gravitational field components (f, g) must also be continuous across a junction and hence $(uf + vg)$ is continuous. The continuity of A therefore involves the continuity of $(\dot{X}\dot{u} + \dot{Y}\dot{v})$, i.e., since $(\dot{u}, \dot{v})$ are continuous

$$\dot{x}_+\dot{u} + \dot{y}_+\dot{v} = \dot{x}_-\dot{u} + \dot{y}_-\dot{v}$$

or

$$(\dot{x}_+ - \dot{x}_-)\dot{u} + (\dot{y}_+ - \dot{y}_-)\dot{v} = 0 \ldots\ldots [56]$$

Equation [54] proves that this condition is equivalent to

$$u\dot{u} + v\dot{v} = 0$$

or

$$\frac{d}{dt}(u^2 + v^2) = 0 \ldots\ldots [57]$$

The quantity $(u^2 + v^2)$ is therefore stationary at the value unity on either side of a junction, and since $u^2 + v^2 \leqq 1$, this stationary value must be a maximum.

To sum up, the conditions to be satisfied at a junction are

(a) (u, v) must be the direction cosines of the thrust,
(b) $(\dot{u}, \dot{v})$ must be continuous,
(c) $(u^2 + v^2)$ must be a maximum, and in addition, clearly,
(d) (x, y) must be continuous. ..... [58]

If the instants of departure and arrival are variable, $A = 0$ at the end points. If the time of transit is specified, A must take the same value at each end point.

At an end point, an impulsive thrust may similarly be shown to be permissible, provided (u, v) can represent at this instant the direction cosines of the thrust.

At a junction between two null-thrust arcs at which no impulse is applied, it is clear that X, Y, $\dot{X}$, $\dot{Y}$ will be continuous and hence X, Y will continue to represent the same solutions of Equations [46], i.e., the two arcs will not be distinct. Further, assuming such a junction, we obtain the first variation of I in the form shown in Equation [20] with

$$L = \left(\frac{\partial H}{\partial t''}\right)_- \quad M = \left(\frac{\partial H}{\partial x''}\right)_- \quad M = \left(\frac{\partial H}{\partial y''}\right)_-$$

$$P = \left(\frac{\partial H}{\partial t''}\right)_+ \quad Q = \left(\frac{\partial H}{\partial x''}\right)_+ \quad R = \left(\frac{\partial H}{\partial y''}\right)_+ \ldots\ldots [59]$$

provided we allow only those variations which do not introduce an impulse at the junction. For such variations, $\delta\dot{x}$, $\delta\dot{y}$ are continuous over the junction (i.e., $\delta\dot{x}_+ = \delta\dot{x}_-$, $\delta\dot{y}_+ = \delta\dot{y}_-$) and since from Equations [59] and [24]–[26] we may write

$$\begin{aligned} &L\delta t_-' + M\delta x_-' + N\delta y_-' - P\delta t_+' - Q\delta x_+' - R\delta y_+' \\ &\quad - \left(\frac{t'\delta x' - x'\delta t'}{t'^2}\right)_+\left(\frac{\partial h}{\partial \dot{x}}\right)_+ - \left(\frac{t'\delta y' - y'\delta t'}{t'^2}\right)_+\left(\frac{\partial h}{\partial \dot{y}}\right)_+ \\ &= \left(\frac{t'\delta x' - x'\delta t'}{t'^2}\right)_-\left(\frac{\partial h}{\partial \dot{x}}\right)_- + \left(\frac{t'\delta y' - y'\delta t'}{t'^2}\right)_-\left(\frac{\partial h}{\partial \dot{y}}\right)_- \\ &= \delta\dot{x}_-\left(\frac{\partial h}{\partial \dot{x}}\right)_- + \delta\dot{y}_-\left(\frac{\partial h}{\partial \dot{y}}\right)_- - \delta\dot{x}_+\left(\frac{\partial h}{\partial \dot{x}}\right)_+ - \delta\dot{y}_+\left(\frac{\partial h}{\partial \dot{y}}\right)_+ \\ &= \delta\dot{x}\left[\left(\frac{\partial h}{\partial \dot{x}}\right)_- - \left(\frac{\partial h}{\partial \dot{x}}\right)_+\right] + \delta\dot{y}\left[\left(\frac{\partial h}{\partial \dot{y}}\right)_- - \left(\frac{\partial h}{\partial \dot{y}}\right)_+\right] \ldots [60] \end{aligned}$$

it follows immediately that $\partial h/\partial\dot{x}$, $\partial h/\partial\dot{y}$ are continuous over the junction. In the limit as $\epsilon \to 0$, these two quantities tend to u and v, respectively, and hence u and v are continuous over the junction. But $\dot{u}$ and $\dot{v}$ are continuous across a junction, and u and v satisfy second-order differential equations. We may therefore conclude that the solutions of these equations on either side of the junction are not distinct. We may accordingly disregard as trivial the possibility of a junction at which no impulse is applied.

This completes the discussion of the minimization of I employing null-thrust arcs and impulsive thrusts applied at their junctions. As has already been mentioned, the nonexistence of minimal trajectories of nonzero thrust has been established in (3). No other possibilities therefore remain to be considered.

3 A First Integral

If neither f nor g depends explicitly on t, i.e., the gravitational field is time invariant, a first integral of Equations [51] is readily obtainable. For, differentiating the equations

$$\ddot{X} + f(X, Y) = 0 \quad \ddot{Y} + g(X, Y) = 0 \ldots\ldots [61]$$

with respect to t, we obtain

$$\dddot{X} + \frac{\partial f}{\partial X}\dot{X} + \frac{\partial f}{\partial Y}\dot{Y} = 0 \quad \dddot{Y} + \frac{\partial g}{\partial X}\dot{X} + \frac{\partial g}{\partial Y}\dot{Y} = 0 \ldots [62]$$

and then, multiplying Equations [51] by $\dot{X}$ and $\dot{Y}$, respectively, and adding, it follows that

$$\dot{X}\ddot{u} + \dot{Y}\ddot{v} - \dddot{X}u - \dddot{Y}v = 0 \quad [63]$$

Integration yields immediately

$$\dot{X}\dot{u} + \dot{Y}\dot{v} - \ddot{X}u - \ddot{Y}v = \text{const.} \quad [64]$$

Employing Equations [61], this first integral may also be written in the form

$$uf + vg + \dot{X}\dot{u} + \dot{Y}\dot{v} = \text{const.} \quad [65]$$

Reference to Equations [55], shows that this latter equation implies that $A = \text{const}$. Since A is continuous across each junction between null-thrust arcs, A must therefore assume the same constant value over the whole trajectory in the case of a time-invariant field. If we are minimizing with respect to a variable time of transit, $A = 0$ at the end points and hence universally.

4 Inverse Square Law Field

In this section we will calculate the form taken by the functions (u, v) when the gravitational attraction is always directed toward a fixed point O of the field and is of magnitude μ/r^2 per unit mass, r being distance measured radially from O. Null-thrust trajectories are then conics with foci at O. Taking Cartesian axes Ox, Oy in the plane of such a trajectory, let (r, θ) be polar coordinates referred to O as pole and Ox as initial line. Then

$$x = r\cos\theta \quad y = r\sin\theta \quad [66]$$

and the trajectory has equation

$$\frac{1}{r} = p\{1 + e\cos(\theta + \gamma)\} \quad [67]$$

where $1/p$ is the semilatus rectum, e is the eccentricity, and γ is an angle determining the orientation of the trajectory in the xy-plane. We have also the angular momentum equation

$$r^2\dot{\theta} = \sqrt{\mu/p} \quad [68]$$

Differentiating Equation [67] with respect to t, we find that

$$\frac{\dot{r}}{r^2} = pe\sin(\theta + \gamma)\,\dot{\theta}$$

and, in view of Equation [68], this can be written

$$\dot{r} = \sqrt{\mu}pe\sin(\theta + \gamma) \quad [69]$$

The quantities f and g are given by the equations

$$f = \frac{\mu}{r^2}\cos\theta = \frac{\mu x}{r^3} = \frac{\mu x}{(x^2 + y^2)^{3/2}}$$

$$g = \frac{\mu}{r^2}\sin\theta = \frac{\mu y}{r^3} = \frac{\mu y}{(x^2 + y^2)^{3/2}} \quad [70]$$

Whence

$$\frac{\partial f}{\partial x} = \mu\frac{y^2 - 2x^2}{(x^2 + y^2)^{5/2}} = \mu\frac{1 - 3\cos^2\theta}{r^3} \quad [71]$$

$$\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x} = -\frac{3\mu xy}{(x^2 + y^2)^{5/2}} = -\frac{3\mu\sin\theta\cos\theta}{r^3} \quad [72]$$

$$\frac{\partial g}{\partial y} = \mu\frac{x^2 - 2y^2}{(x^2 + y^2)^{5/2}} = \mu\frac{1 - 3\sin^2\theta}{r^3} \quad [73]$$

and these quantities may now be substituted in Equations [51], it being understood that r is given in terms of θ by Equation [67].

Clearly θ is more suitable as the independent variable of Equations [51] than is t in this context. Using primes to denote differentiations with respect to θ throughout this section, we have

$$\ddot{v} = v''\dot{\theta}^2 + v'\ddot{\theta} \quad [74]$$

Equation [68] gives

$$\dot{\theta} = \frac{1}{r^2}\sqrt{\frac{\mu}{p}} \quad [75]$$

and this, upon differentiation with respect to t, leads to

$$\ddot{\theta} = -\frac{2}{r^3}\dot{r}\sqrt{\frac{\mu}{p}} = -\frac{2\mu e}{r^3}\sin(\theta + \gamma) \quad [76]$$

having employed Equation [69].

Equation [74] can now be written

$$\ddot{v} = \frac{\mu}{pr^4}v'' - \frac{2\mu e}{r^3}v'\sin(\theta + \gamma) \quad [77]$$

and hence the second of Equations [51] is equivalent to

$$\{1 + e\cos(\theta + \gamma)\}v'' - 2ev'\sin(\theta + \gamma) - 3u\sin\theta\cos\theta + v(1 - 3\sin^2\theta) = 0 \quad [78]$$

Simplification of this equation may be effected by changing to new dependent variables (w, z), related to the variables (u, v) according to the equations

$$w = (u\cos\theta + v\sin\theta)\{1 + e\cos(\theta + \gamma)\} \quad [79]$$

$$z = (-u\sin\theta + v\cos\theta)\{1 + e\cos(\theta + \gamma)\} \quad [80]$$

In terms of w and z, Equation [78] will be found to take the form

$$z'' + 2w' = 0 \quad [81]$$

Equation [81] integrates immediately to yield

$$z' = Ee - 2w \quad [82]$$

E being an arbitrary constant.

Since f and g do not depend explicitly upon t, Equation [64] is available.

Differentiating the equation $x = r\cos\theta$ with respect to t

$$\begin{aligned}\dot{x} &= \dot{r}\cos\theta - r\dot{\theta}\sin\theta \\ &= \sqrt{\mu p}\,e\sin(\theta + \gamma)\cos\theta \\ &\quad - \sqrt{\mu p}\{1 + e\cos(\theta + \gamma)\}\sin\theta \\ &= \sqrt{\mu p}(e\sin\gamma - \sin\theta)\end{aligned} \quad [83]$$

Equations [67] and [69] having been employed in the reduction.

Differentiating Equation [83] again, we obtain

$$\ddot{x} = -\sqrt{\mu p}\cos\theta\,\dot{\theta} \quad [84]$$

Similarly we may show that since $y = r\sin\theta$

$$\dot{y} = \sqrt{\mu p}(e\cos\gamma + \cos\theta) \quad [85]$$

$$\ddot{y} = -\sqrt{\mu p}\sin\theta\dot{\theta} \quad [86]$$

Equation [64] is now seen to take the form

$$(e\sin\gamma - \sin\theta)u' + (e\cos\gamma + \cos\theta)v' + u\cos\theta + v\sin\theta = 0 \quad [87]$$

the constant on the right-hand side being given the value zero in order that minimization may be effected with respect to transit time.

Transforming to the variables w and z as defined by Equations [79] and [80], this latter equation becomes

$$e(1 + e\cos\psi)\sin\psi\,w' + (1 + e\cos\psi)^2 z' + (e^2 + 2 + 3e\cos\psi)w = 0 \quad [88]$$

where ψ has been written for $\theta + \gamma$.

Equations [82] and [88] specify between them the functions w and z. Elimination of z' between them provides us with a first-order linear equation for w, viz.,

$$(1 + e\cos\psi)\sin\psi\, w' + (e - \cos\psi - 2e\cos^2\psi)w = -E(1 + e\cos\psi)^2 \quad [89]$$

The integrating factor is found to be $(1 + e\cos\psi)^{-2}\sin^{-2}\psi$ and the solution for w accordingly

$$w = (1 + e\cos\psi)(De\sin\psi + E\cos\psi) \quad [90]$$

where D is a second arbitrary constant.

Substituting from Equation [90] into Equation [82], we obtain upon integration for z

$$z = D(1 + e\cos\psi)^2 - E\sin\psi(2 + e\cos\psi) + F \quad [91]$$

F being a third constant of integration.

We now introduce new dependent variables (U, V) defined by the equations

$$U = \frac{w}{1 + e\cos\psi} = u\cos\theta + v\sin\theta$$

$$V = \frac{z}{1 + e\cos\psi} = -u\sin\theta + v\cos\theta \quad [92]$$

Equations [90] and [91] can then be written

$$U = De\sin\psi + E\cos\psi \quad [93]$$

$$V = D(1 + e\cos\psi) - E\sin\psi + \frac{F - E\sin\psi}{1 + e\cos\psi} \quad [94]$$

If $O'u$, $O'v$ are rectangular Cartesian axes in a plane, we may interpret the quantities (u, v) as the resolutes of a certain displacement vector $O'P$. Equations [92] then show that (U, V) are to be interpreted as the resolutes of the same vector with respect to axes $O'U$, $O'V$ making an angle θ with the axes $O'u$, $O'v$ as indicated in Fig. 1. At a junction, (u, v)

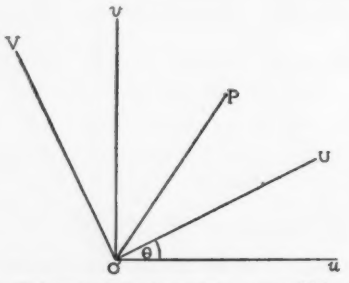


FIG. 1 RELATIONSHIP BETWEEN uv- AND UV-PLANES

represent the direction cosines of the thrust direction, and hence, if we suppose $O'u$, $O'v$ to be in the directions of the axes Ox, Oy, respectively, the vector $O'P$ will then be a unit vector in the direction of the thrust. Also, $O'U$, $O'V$ will be in the directions of the radius vector r and the perpendicular to this line, respectively. (U, V) are accordingly the resolutes of the unit vector in the direction of the thrust, taken along the radial and transverse directions. If, therefore, ϕ is the angle made by the direction of the thrust with the forward transverse direction, the following equations are valid at a junction

$$U = \sin\phi \quad V = \cos\phi \quad [95]$$

Differentiating Equations [92] with respect to t, we obtain

$$\dot{U} = V\dot{\theta} + \dot{u}\cos\theta + \dot{v}\sin\theta$$
$$\dot{V} = -U\dot{\theta} - \dot{u}\sin\theta + \dot{v}\cos\theta \quad [96]$$

or, employing Equation [75]

$$\sqrt{\frac{\mu}{p}}(U' - V) = (\dot{u}\cos\theta + \dot{v}\sin\theta)r^2$$

$$\sqrt{\frac{\mu}{p}}(V' + U) = (-\dot{u}\sin\theta + \dot{v}\cos\theta)r^2 \quad [97]$$

But $\dot{u}$, $\dot{v}$, r, and θ are to be continuous across a junction for a minimal trajectory, and it therefore follows that the quantities

$$\frac{1}{p^{1/2}}(U' - V), \quad \frac{1}{p^{1/2}}(V' + U) \quad [98]$$

are also continuous across a junction.

We can now write down all the conditions to be satisfied by an optimal trajectory in an inverse square law field. The resulting equations are of practical importance, since the major part of the track of any future interplanetary rocket will lie in such a field.

5 Transfer Between Two Similar Orbits

The problem of optimal transfer of a rocket between two coplanar orbits of identical sizes and shapes and described in the same sense, but differing as regards their orientation, has been solved in (1). This exercise will now be employed to test the accuracy of the theory developed in this paper. The method of attack used in (1) was quite distinct from that advocated here.

Taking the elements of the orbits to be (p, e, γ), $(p, e, -\gamma)$, so that the initial line is a line of symmetry, we shall reduce the labor, as in (1), by assuming the trajectory of transfer also to lie symmetrically with respect to the two terminal trajectories (see Fig. 2). The elements of the transfer orbit will therefore be taken as $(p', e', 0)$. Impulses are applied at P and P' to effect transfer from a terminal orbit into the orbit of transfer and from the latter orbit into the second terminal orbit. Let P have polar coordinates (r, θ). Then P' has coordinates $(r, -\theta)$. It is shown in (3) that, during the short duration of an impulsive thrust applied to a rocket at the point (r, θ) in a conic orbit, the elements (p, e, γ) of the rocket's orbit, vary according to the equations

$$pe\cos(\theta + \gamma) = s - p \quad [99]$$

$$pe\sin(\theta + \gamma) = (s - p^{1/2}W)\tan\phi \quad [100]$$

where $s = 1/r$, ϕ is the angle defined in the previous section fixing the direction of thrust, and $\mu^{1/2}W\sin\phi$ is the component

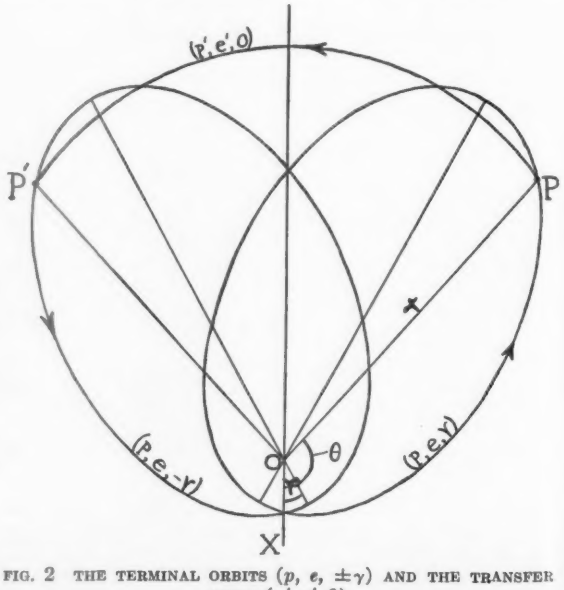


FIG. 2 THE TERMINAL ORBITS $(p, e, \pm\gamma)$ AND THE TRANSFER ORBIT $(p', e', 0)$

of the rocket's velocity in a direction normal to the thrust. This latter component is unaffected by the thrust. If (ϕ, W) correspond to the thrust at the point P, we have, in view of these equations

$$pe \cos \psi = s - p \quad [101]$$

$$pe \sin \psi = (s - p'^{1/2}W) \tan \phi \quad [102]$$

$$p'e' \cos \theta = s - p' \quad [103]$$

$$p'e' \sin \theta = (s - p'^{1/2}W) \tan \phi \quad [104]$$

where $\psi = \theta + \gamma$. On account of the symmetry, the thrust at P' is characterized by parameters $(\pi - \phi, W)$ and it may be verified that the above equations may also be obtained by consideration of the thrust at P'.

Let the three arbitrary constants associated with the functions (U, V) in Equations [93] and [94], take the values (D, E, F) while the rocket orbits in the ellipse (p, e, γ) take the values (D', E', F') in the transfer orbit. We shall find that all our minimizing conditions can be satisfied by taking the values of these quantities in the final orbit to be $(-D, E, -F)$. This is accounted for by the existence of symmetry.

Equations [95] will now be found to be satisfied at the junctions P and P' provided

$$De \sin \psi + E \cos \psi = D'e' \sin \theta + E' \cos \theta = \sin \phi$$

$$D(1 + e \cos \psi) - E \sin \psi + \frac{F - E \sin \psi}{1 + e \cos \psi} = D'(1 + e' \cos \theta) - E' \sin \theta + \frac{F' - E' \sin \theta}{1 + e' \cos \theta} = \cos \phi$$

$$-D'e' \sin \theta + E' \cos \theta = \sin \phi$$

$$-D' (1 + e' \cos \theta) - E' \sin \theta - \frac{F' + E' \sin \theta}{1 + e' \cos \theta} = \cos \phi \quad [105]$$

It is now clear that $D' = F' = 0$, and hence these equations reduce to

$$De \sin \psi + E \cos \psi = E' \cos \theta = \sin \phi \quad [106]$$

$$D(1 + e \cos \psi) - E \sin \psi + \frac{F - E \sin \psi}{1 + e \cos \psi} = -E' \sin \theta \frac{2 + e' \cos \theta}{1 + e' \cos \theta} = \cos \phi \quad [107]$$

The quantities [98] being continuous across a junction, there are also the equations

$$\frac{1}{p^{1/2}} \left(\frac{E \sin \psi - F}{1 + e \cos \psi} - D \right) = \frac{1}{p'^{1/2}} \cdot \frac{E' \sin \theta}{1 + e' \cos \theta} \quad [108]$$

$$\frac{1}{p^{1/2}} \left[\frac{Ee + E \cos \psi - Fe \sin \psi}{(1 + e \cos \psi)^2} \right] = \frac{1}{p'^{1/2}} \cdot \frac{E'(e' + \cos \theta)}{(1 + e' \cos \theta)^2} \quad [109]$$

Equations [101]–[104], [106]–[109] are ten in number and are therefore sufficient to determine the ten unknowns $D, E, F, E', \phi, W, \theta, s, p', e'$ and hence to solve the problem. We shall now eliminate D, E, F, E' to yield finally Equations [101]–[104] and two others, as the solution to our problem. This stage has already been reached in (1), where a method for the solution of these six equations is also given.

Eliminating F between Equations [108] and [109], we obtain

$$\frac{De \sin \psi + E \cos \psi}{E' \cos \theta} = \frac{\dfrac{p^{1/2}}{p'^{1/2}} (e' + \cos \theta)(1 + e \cos \psi) - e \sin \theta \sin \psi (1 + e' \cos \theta)}{\cos \theta (1 + e' \cos \theta)^2}$$

Equations [106] show that this is equivalent to

$$(e' + \cos \theta)(1 + e \cos \psi) - e \sin \theta \sin \psi (1 + e' \cos \theta) = \frac{p'^{1/2}}{p^{1/2}} \cos \theta (1 + e' \cos \theta)^2 \quad [110]$$

Elimination of E' between Equations [106] and [107] gives

$$\tan \phi \tan \theta = -\frac{1 + e' \cos \theta}{2 + e' \cos \theta} \quad [111]$$

Equations [110], [111] represent the result of eliminating D, E, F, E'. Employing Equations [101]–[104] we may put them in the form derived in (1).

Squaring and adding Equations [103] and [104], we get

$$p'^2e'^2 = (s - p')^2 + (s - p'^{1/2}W)^2 \tan^2 \phi \quad [112]$$

Division of these two equations also gives

$$\tan \theta = \frac{s - p'^{1/2}W}{s - p'} \tan \phi \quad [113]$$

Equation [103] may be written

$$1 + e' \cos \theta = \frac{s}{p'} \quad [114]$$

Equations [111], [113], and [114] now yield

$$\tan^2 \phi = \frac{s(p' - s)}{(p' + s)(s - p'^{1/2}W)} \quad [115]$$

Substituting for $\tan^2 \phi$ in Equation [112], we find that

$$e'^2 = \frac{(s - p')(Ws - p'^{3/2})}{p'^{3/2}(s + p')} \quad [116]$$

From the last equation and Equation [114] we now deduce that

$$e'^2 + e' \cos \theta = \frac{s(s - p')(p'^{1/2} + W)}{p'^{3/2}(s + p')} \quad [117]$$

Employing Equations [102], [104], [114], and [115], we may also show that

$$ee' \sin \theta \sin \psi (1 + e' \cos \theta) = \frac{s^2(p' - s)(s - p'^{1/2}W)}{pp'^2(p' + s)} \quad [118]$$

Multiplying Equation [110] throughout by e' and employing Equation [114], it may be put in the form

$$(e'^2 + e' \cos \theta) \frac{s}{p} - ee' \sin \theta \sin \psi (1 + e' \cos \theta) = \frac{s^2(s - p')}{p'^{1/2}p^{3/2}} \quad [119]$$

Equations [117] and [118] show that this is equivalent to

$$(s + p' + p'^{1/2}W)(p'^{1/2} - p^{1/2}) = 0$$

or

$$s + p' + p'^{1/2}W = 0 \quad [120]$$

This latter equation is a simplified form of Equation [110]. Equation [111] has been shown to be equivalent to Equation [115], and this, in turn, may be shown to be equivalent to

$$\left(1 + \frac{p'^{1/2}}{W}\right)(s - p') + (s - p'^{1/2}W) \tan^2 \phi = 0 \quad [121]$$

by use of Equation [120].

Equations [101]–[104], [120], [121] will be found quoted in Ref. (1).

(Continued on page 382)

The Stability of Ethylene Oxide

E. MILTON WILSON[1]

Aerojet-General Corporation, Azusa, Calif.

The stability of ethylene oxide under adverse pressurization conditions has been demonstrated by a series of tests in which the compound was pressurized at rates as great as 200,000 psi/sec and was ejected through small, sharp-edge orifices at high pressure drops. No detonations occurred in these tests, or even when the material was suddenly pressurized with oxygen and ejected against a stainless steel block maintained at 1000 F.

Introduction

IT IS an inherent property of all compounds with a positive heat of decomposition that some degree of instability will be exhibited upon the application of an external source of energy such as heat or shock waves. Although the normal hazards in handling ethylene oxide have been adequately reported (1, 2),[2] data on the sensitivity under two conditions peculiar to the use of this material under more adverse handling conditions have not yet been published. The two unknown characteristics were its sensitivity during rapid pressurization and during ejection through sharp-edged orifices; since either of these conditions is capable of producing a serious detonation, tests were made in which extremes of pressure were used, so that marginal stability could be readily detected.

Testing Methods

The stainless steel bomb shown in Fig. 1 was used both for tests at high rates of pressurization and ejection tests. It

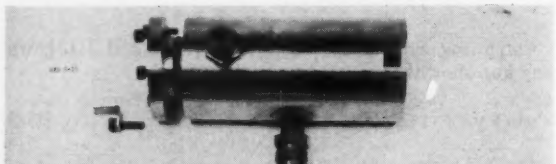


FIG. 1 SMALL PRESSURIZATION TESTER, DISASSEMBLED

was desired to place a maximum pressure of 20,000 psi upon the sample of ethylene oxide, but as no direct means of imparting so high a pressure is readily available, it was necessary to use a multiplying-piston arrangement with an area ratio of 20:1, thus making it possible to achieve a pressure of 20,000 psi on the small end of the piston by applying the pressure obtainable from a compressed-nitrogen cylinder to the large end. The design features and critical dimensions of the tester are: Volume of sample cell, 2.0 cc; neoprene O-ring seals on all sliding surfaces and on the top closure in the nitrogen chamber; diameter of sample chamber, 0.50 in.; diameter of nitrogen-pressure chamber, 2.45 in.; minimum wall thickness of sample chamber, 0.58 in.; maximum stroke of piston, 0.56 in.

High rates of pressure rise were achieved by the sudden admission of compressed nitrogen to the low-pressure chamber of the tester. A maximum pressurization rate of approximately 15,000 psi/sec was thus achieved in the low-pressure chamber, resulting in a twentyfold pressurization rate (approximately 300,000 psi/sec) in the high-pressure chamber as a result of the multiplier system. Rapid admission of the compressed nitrogen was accomplished by means of a solenoid-operated, four-way Saval valve equipped with a variable orifice on its outlet side, the rate of pressure increase being determined by a 2000-psi variable-reluctance pickup attached through a tee to the nitrogen-chamber inlet line. This pickup, when used in conjunction with an oscillograph, provided a permanent record of the rate of pressurization. The inlet side of the four-way valve was supplied from a high-pressure nitrogen manifold which was controlled through a Grove pressure regulator. A schematic diagram of the system is shown in Fig. 2.

Although the ratio of the area of the large end of the piston to the area of the small end was designed to be 20, slight variations in the machining of each piston as well as friction effects existing in a dynamic system made it necessary to calibrate each tester to obtain the effective area ratio. Calibration was performed by pressurizing the sample side of the tester with a hydraulic pump and determining the pressure on the nitrogen-chamber side necessary to cause the piston to move suddenly against this applied hydraulic pressure; a plot of one gage reading against the other at a series of pressures results in a straight line whose slope is the desired ratio. Tests in which electronic pickups were present on both high-pressure and low-pressure sides indicated the reliability of determining rates of pressurization in this manner.

Experimental

A freshly cleaned and lubricated bomb was chilled with liquid nitrogen so that the ethylene oxide (bp 51 F) would have a low vapor pressure when introduced. A 0.8- to 1.2-cc quantity of precooled material was then added and sealed

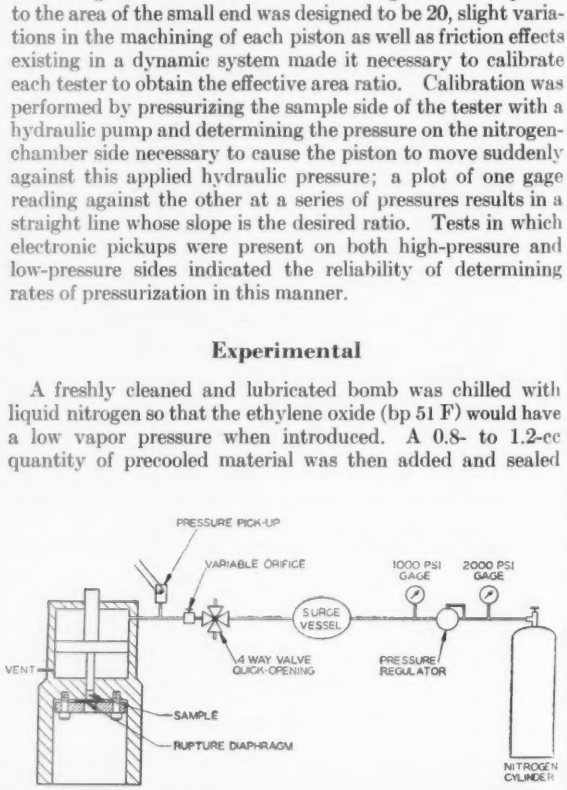


FIG. 2 SCHEMATIC DIAGRAM OF PRESSURIZATION SYSTEM

Received July 25, 1953.

[1] Chemist, Solid Engine and Chemical Division. Member ARS.

[2] Numbers in parentheses refer to References on page 369.

TABLE 1 SUMMARY OF TEST DATA

Distribution of 526 pressurization tests

System	Rate of pressurization, psi/sec: <50,000 Amb.	<50,000 200 F	50,000–150,000 Amb.	50,000–150,000 200 F	150,000 Amb.	150,000 200 F
Ethylene oxide + O_2	28	46	62	52	1	2
Ethylene oxide + air	46	17	98	13	15	2
Ethylene oxide + its own vapor	41	3	43	12	1	1
Ethylene oxide + air + rust	3	..	8	..	..	..
Ethylene oxide + air + 1 drop CH_3COOH	3	..	8	..	..	..
Ethylene oxide + air + $FeCl_3$	3	..	8	..	..	..
Ethylene oxide + air + NaOH	3	..	7	..	..	..

Distribution of 115 ejection tests

Pressure drop, psi	Orifice diam, in., at following temperature: 0.005, 150 F	0.020, 150 F	0.032, 52 F	0.032, 150 F
5,000	10	10	5	10
12,000	3	5	..	1
15,000	10	10	7	10
20,000	10	10	3	10

Test conditions in heated-block ejection tests

Block temp., ° F	Oxygen pressure, psig	Ethylene oxide, Temp., ° F
400	800	70
400	800	150
700	800	150
700	800	150
700	1000	150
700	1000	150
1000	1000	150

within the bomb by forcing, by means of a collar and bolts, a 0.125-in.-diam aluminum disk against steel serrations which were concentric with the sample cylinder. The ethylene oxide was retained within the tester by this burst diaphragm, by extremely close tolerances between piston and cylinder, and by an O-ring on the piston. When it was desired to test the sensitivity to compression in the presence of air, the sample was sealed immediately after introduction of the ethylene oxide; if oxygen was to be the atmosphere, a gaseous oxygen purge was used as the sample was added; and if no diluent atmosphere was desired, the ethylene oxide was allowed to boil before a seal was made.

The bomb was heated in a water bath to the desired temperature, and a pressure between 100 and 1000 psi was set on the Grove regulator. The solenoid-operated valve was then opened and the rate of pressure rise was recorded on the oscillograph. The rate was set approximately by adjusting the hole size of the orifice on the downstream side of the valve, giving a range of rates dependent on the applied pressure.

The sensitivity to passage through small orifices was tested by forcing ethylene oxide through orifices of 0.032-in., 0.020-in., and 0.005-in. diam at pressure drops of 5000, 15,000, and 20,000 psi. The pressurization bomb was utilized for these tests by clamping a steel orifice plate between an aluminum sealing disk and a bolted collar. Because the tests were made at 150 F, at which temperature the vapor pressure of ethylene oxide is approximately 90 psia, it was necessary to seal the liquid within the cylinder by means of a 0.001-in.-diam aluminum rupture disk. Expulsion was accomplished by releasing nitrogen pressure from the solenoid-operated valve through a short, unrestricted line connected to the large chamber of the tester.

Gupta (3) has demonstrated that several ethylene oxide explosions were probably caused by certain impurities which catalyze polycondensation of the material. Several of these common impurities were added to ethylene oxide to determine the effect upon the sensitivity to compression; acetic acid, sodium hydroxide, ferric chloride, or rust did not alter the stability of the oxide.

As it is conceivable that a fast-moving stream of ethylene oxide might impinge against warm surfaces, ethylene oxide was ejected against a heated stainless-steel block to observe the effect. A U-tube, two thirds filled with ethylene oxide (15 cc) was suddenly pressurized with oxygen, and the liquid was ejected through one of the steel orifice plates. Even with the fuel at 150 F and the block at 1000 F, no reaction of the oxide could be observed.

At intervals during the testing, nitromethane was used in place of ethylene oxide, as it was known that this compound would detonate in the presence of air at rates of pressurization less than 50,000 psi/sec; a check on the design of the equipment was thus possible. No difficulty was ever encountered in causing detonation of nitromethane samples at pressures substantially less than those applied on ethylene oxide.

Results

A summary of the tests performed is presented in Table 1. As no detonations occurred in any of the tests, it is impossible to make a statistical analysis of the results, and for this reason the data are presented merely according to the type of test and test conditions. The values in the table indicate the number of tests performed under each of the specified conditions.

Acknowledgment

The design of the pressurization bomb is that of Mr. B. B. Mandell. The author takes pleasure in expressing his appreciation for the use of this design and for the assistance received during this study. The financial support of the Office of Naval Research is also gratefully acknowledged.

References

1 "Properties and Essential Information for Safe Handling and Use of Ethylene Oxide," Chemical Safety Data Sheet SD-38, 1951, Manufacturing Chemists' Association, 246 Woodward Bldg., Washington 5, D. C.

2 "Ethylene Oxide, Hazards and Methods of Handling," by L. G. Hess and V. V. Tilton, *Industrial and Engineering Chemistry*, vol. 42, 1950, pp. 1251–1258.

3 "The Explosive Poly-Condensation of Ethylene Oxide," by A. K. Gupta, *Journal of the Society of Chemical Industry (London)*, vol. 68, 1949, pp. 179–183.

Technical Notes

Scintillations of Stars and Indirect Meteorology[1]

F. ZWICKY[2]

California Institute of Technology, Pasadena, Calif.
Aerojet-General Corporation, Azusa, Calif.

ATMOSPHERIC disturbances of various kinds produce scintillations (twinkling) of the stars and "boiling" of the images of distant extended objects as seen in telescopes and in field glasses(1).[3] The photography of many celestial objects and in particular the study of the surface features of the moon and of the planets have been greatly hampered because of the interference of fluctuations in density and in refractive index of various parts of the earth's atmosphere. The photography and the tracking of high or far-flying rockets likewise yield poor results because the contours of the projectiles are seen to flicker with excursions which generally amount to several seconds of arc. Two problems immediately arise in connection with the optical effects caused by atmospheric disturbances. These are: 1 Is it possible to eliminate or to compensate for the imperfections in optical image formations which are due to the unsteadiness of the atmosphere? 2 Is it possible to make constructive uses of the study of optical scintillation effects? The answer to both of these questions is essentially in the affirmative. Before complete answers can be given, however, much work remains to be done on the nature of atmospheric fluctuations. Some basic results have recently been achieved with telescopes whose apertures range from 3 inches to 200 inches (2). We here present a few of these results which point the way to how one is to proceed if he wishes to achieve an over-all continuous and highly time-resolved survey of the various disturbances in the earth's atmosphere.

The present discussion is restricted to the consideration of certain disturbances such as vortices which are carried along by horizontal winds.

1 Drifted Extrafocal Star Images

Scintillations of stars have been watched by countless astronomical observers on still more countless occasions. Only very little work has been done, however, to analyze the quantitative aspects of the phenomena involved. For such analyses generally the stationary or the drifted in-focus images of stars were observed and their lateral displacements and intensity variations were recorded (1). The writer has found that far more information can be gained if either *drifted extrafocal star images* or *drifted focused spectra* are photographed and analyzed.

Suppose that light from a star is focused at A by a lens or mirror of aperture D and focal length f. (See Fig. 1a.) A vortex or eddy E in the atmosphere which passes through the beam of parallel light from the star to the telescope may act as a positive or negative lens and focus at B a part of the light contained in the beam of width D. An instantaneous exposure of a photographic plate located extrafocally at B will thus show a continuously illuminated circular surface, with an extra point of light at B. If the vortex has velocity components V_W and V_N relative to the lens, toward the west and the north, respectively, the point B in a stationary telescope will move on the photographic plate with the velocity components $v_W = V_W\Delta f/f + u$, and $v_N = V_N\Delta f/f$, respectively, where $u = \Omega \times (f + H\Delta f/f) \sin \chi$ is the term which is due to the rotation of the earth relative to the stars with the angular velocity Ω. The angle χ is given by

$$\cos \chi = \sin \Lambda \cos Z - \cos \Lambda \sin Z \cos h$$

where h is the hour angle at which the telescope is stationary. Δf = distance between the focal plane and the parallel plane through B; Λ = the latitude of the location of the telescope; Z = the zenith distance (angle); and H = the height of the vortex above the telescope.

In order to time the various events which are being recorded, either the telescope may be turned successively and at uniform angular rates west and north, or, in two operations, the photographic plate may be moved in the stationary telescope east and south with the velocities w_E and w_S. The latter procedure is chosen. On exposing and moving the plate, the original out-of-focus circle will be drawn out into two strips which are oriented east-west and north-south (Fig. 1b). The two bright lines BB′ and BB″ indicate the paths of the extrafocal point image B. If there exists an appreciable vertical velocity component V_z of the vortex E, the image B will appear drawn out into a long thin wedge, rather than a line of more or less uniform thickness. The order of magnitude of V_z can be estimated from the angle of these wedges.

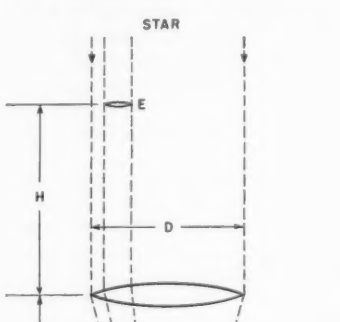


FIG. 1A INTERFERENCE OF AN ATMOSPHERIC VORTEX WITH THE FOCUSING OF LIGHT FROM A STAR BY A LENS

[1] Paper presented at a meeting of the Southern California Section of the AMERICAN ROCKET SOCIETY, Los Angeles, Calif., September 24, 1953.

[2] Professor of Astrophysics, California Institute of Technology; Research Consultant, Aerojet-General Corporation. Fellow ARS.

[3] Numbers in parentheses refer to Bibliography at end of paper.

EDITOR'S NOTE: This section of the JOURNAL is open to short manuscripts describing new developments or offering comments on papers previously published. Such manuscripts are published without editorial review, usually within two months of the date of receipt. Requirements as to style are the same as for regular contributions.

Returning to the lines BB′ and BB″, their inclinations α and β relative to the two directions of slewing used are given by

$$\tan \alpha = v_N/(v_W + w_E) = V_N \Delta f/(V_W \Delta f + uf + w_E f) \ldots [1]$$

and

$$\tan \beta = v_W/(v_N + w_S) = (V_W \Delta f + uf)/(V_N \Delta f + w_S f) \ldots [2]$$

The unknowns in these equations are V_N, V_W, and H. From Equations [1] and [2] we can determine V_N but not V_W and H. If we move our photographic plate with a velocity $w_E \gg v_W$ we obtain V_N from [1] alone. Later on an entirely different set of observations will be described which determine H once V_N is known. All of the three unknowns V_N, V_W, and H can, in principle, be determined from [1] and [2], if two different rates of either w_E or w_S are used successively and if Z or Δf are changed likewise. Actually one may exploit the relations [1] and [2] in a number of ways. One only need vary w_E, w_S, Z, and Δf (use of stepped photographic

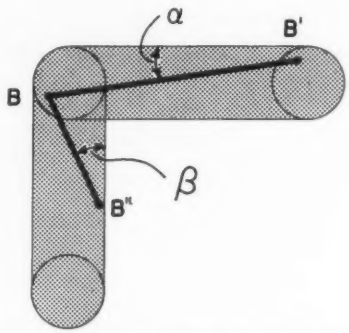


FIG. 1B DRIFTED EXTRAFOCAL STAR IMAGES (EAST-WEST AND SOUTH-NORTH) WITH STRIATIONS DUE TO THE INTERFERENCE OF ATMOSPHERIC VORTICES

plates) either during one observation or a series of observations. If one specific vortex does not last long enough, one can observe a succession of vortices which always follow one another in the same wind strata. Much can be gained with the method of drifted extrafocal images if narrow band color filters are used.

2 Drifted Focused Spectra

A telescope which is equipped with a full-sized objective prism or grating oriented north-south will draw out each stellar image on the focal surface into a thin line showing the various spectral features of the star. If the telescope is stationary or is turning with a uniform angular velocity different from that of the earth's rotation, the spectrum will be continuously drawn out along a time axis t. If the atmosphere were perfectly steady the widened spectrum would appear as a uniform band with the spectral lines running straight along it. Scintillations cause fluctuations of intensity along the time axis which will be different for different colors. Also the spectral lines will not run straight but show both over-all and differential displacements along the direction of color dispersion. (See Fig. 2a). Astronomical "seeing" can be better in one color range than in another. A great amount of information can be derived from drifted spectra. Here we are only concerned with one systematic feature which makes possible the determination of the height H at which a vortex passes the observed beam of light. Consider again the vortex E which is drifting north with the velocity component V_N. Note that rays of different color (red and violet, for instance) from a given star, which reach the point of observation S on the surface of the earth simultaneously, are not coincident in space. (See Fig. 2b.)

Because of the refraction within the atmosphere, rays of different wave lengths are separated by horizontal distance Δ which increases with H and Z. For instance, for the zenith "distance" $Z = 50$ deg, the rays λ 4000 A and λ 6800 A are separated by $\Delta_0 = 7.7$ cm at the top of the atmosphere. A vortex E moving north intercepts the red rays of a star south of zenith first, say at the time t_0, and reaches the violet rays at the time t. It thus is $t - t_0 = \Delta/V_N$. The value of V_N was already derived from the observations described in section 1. On the other hand, $t - t_0$ can be determined from the inspection of the drifted spectrum. Indeed, at the time t_0 when the vortex reaches the red ray, the surface brightness in the focal plane is weakened at the red end of the spectrum, because the vortex throws some of the red light out of focus. As the vortex passes through the beam collected by the telescope, this spot of decreased intensity runs diagonally through the drifted spectrum from the longer to the shorter waves and reaches the violet end at the time t. From the observed difference $t - t_0$ we may thus calculate $\Delta = (t - t_0)V_N$. Since Δ is known as a function of H and z, the height H of the vortex responsible for the diagonal band in the drifted spectrum may thus be derived from the observations which were described. If vortices are carried by winds of different speeds or at different heights, several sets of weak and strong intensity bands make their appearance in the drifted spectra. Their velocities and locations can be obtained simultaneously, using the method described. A minimum of three photographs is necessary, namely exposures of two drifted extrafocal images of one star and a drifted focused spectrum of the same star. Through the application of various rates of the imposed drifts, a number of observations can be made from all of which the same three unknowns V_W, V_N, and H can be determined.

3 Future Observations

Proceeding systematically with the analysis of drifted extrafocal star images and drifted but sharply focused spectra, a wealth of information can be obtained. This information includes data on the size of the vortices in the atmosphere, their statistical distribution, their optical thickness (geometrical thickness multiplied with the change of air density within the vortex), characteristics of pressure waves and shock waves which arise from thunderclaps and other sources. With large telescopes and very rapid motions of the photo-

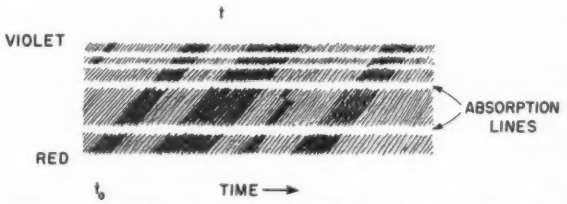


FIG. 2A STRIATIONS IN THE DRIFTED FOCUSED SPECTRUM OF A STAR WHICH ARE CAUSED BY ATMOSPHERIC DISTURBANCES SOUTH OF THE ZENITH MOVING SOUTH-NORTH

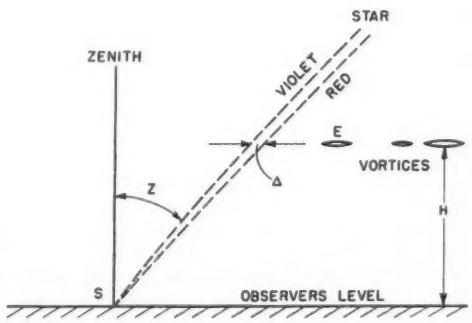


FIG. 2B SEPARATION OF LIGHT RAYS OF DIFFERENT COLORS FROM A STAR BECAUSE OF REFRACTION IN THE ATMOSPHERE

graphic plates or rapid slewing of the telescopes, time resolutions can be obtained which are better than 10^{-6} seconds. It is rather surprising to find that there are large changes (several powers of ten (in the light from stars entering medium-sized telescopes even within these short intervals, especially if narrow spectra bands are considered. Also, the observations of the fast drifted spectra prove for the first time that, because of the observed inclinations of the intensity bands in the drifted spectra, atmospheric disturbances at all heights up to many kilometers contribute materially to the scintillations of star light. In order to check these findings directly, the author has been able to arrange for flights of planes at heights of 3, 6, 9, and 12 kilometers on which the methods described in this paper were used. The flights are being continued.

It has been proposed that some of the effects of poor seeing could be eliminated if photoelectronic telescopes were used with image stabilization on the second focal surface (3). It appears possible in this fashion to stabilize the image of a single star to a considerable extent. The question arises, however, as to the possibility of stabilizing a sizable surface image such as that of a planet or part of the moon. The answer to this question depends on the degree to which two neighboring celestial sources of light are subject to scintillations which are in phase with one another. Experiments which the author has carried out on double stars whose separations amount to from three seconds of arc to 62 seconds of arc (ν' and ν^2 Draconis) have shown that the displacements of the two stars in many cases of medium seeing are indeed correlated to a remarkable extent. It would thus appear that the stabilization of a single point of the edge of a planet, for instance, would stabilize the whole disk sufficiently often and sufficiently long (fraction of a second) to yield some perfect photographs in a series of exposures. Any results of this kind, once they can be realized with the photoelectronic telescope, will be of importance also for the complete tracking of all sorts of missiles along their trajectories. A first surprisingly successful version of the photoelectronic telescope has recently been constructed by A. Lallemand (4) who is one of the pioneers in this field.

4 Indirect Meteorology

The results obtained from observations of astronomical "seeing" both at night and in daytime (on the sun and on bright stars) promise to make possible the indirect monitoring of the clear or partly clear atmosphere. The data, in collaboration with the meteorologists, will have to be correlated with weather data, jet streams, signals from distant storms, and so on. Once these correlations are established there is hope that many of the observational techniques using balloons, rockets, and planes can be dispensed with and replaced by inexpensive optical methods. The plans for the present are that all of the major optical phenomena involved will be analyzed with methods similar to those described in this paper. An attempt will then be made to organize some of the amateur astronomers and perhaps a few professional astronomers to make simultaneous observations of stellar scintillations over the whole continent and to correlate the results with weather data. In this program the staff of *Scientific American* has kindly consented to lend a helping hand.

5 Generalization of the Methods Described

Important generalizations of the problems here discussed immediately follow if we consider the transition from a clear atmosphere to one in which moisture is precipitated in various locations. Also, instrumentwise, instead of using a single telescope the observations can be strengthened through simultaneous tests with two or more telescopes. This introduces, in particular, the elements of triangulation. The mounting of the telescopes or cameras after the fashion of theodolites, with one vertical and one horizontal axis of rotation, will be most advantageous for this type of work. Useful results can be obtained with lenses of a few inches aperture and 10 to 20 inches focal length, especially if they can be equipped with full-sized replica gratings or prisms. Last but not least, telescopes and receivers sensitive to entirely different wave-length ranges can be used. Some work actually has already been done with short and long radio waves. Finally, the twinkling of sources of sound may be advantageously investigated with analogous methods.

Bibliography

1 For a complete Bibliography see Folke Nettelblad, "Studies of Astronomical Scintillation," Publications of the Lund (Sweden) Observatory, Series II, no. 130, 1953.

2 "Seeing," by F. Zwicky, Publications of the Astronomical Society of the Pacific, 1950, vol. 62, p. 150.

3 "Morphological Astronomy," by F. Zwicky (Halley Lecture at Oxford University, May 12, 1948), *Observatory*, 1948, vol. 68, p. 845.

4 "Application a l'astronomie d'un recepteur ideal de photons," by A. Lallemand and M. Duchesne, *Comptes Rendus*, 1952, vol. 235, p. 503.

Take-Off From Satellite Orbits

KRAFFT A. EHRICKE[1]

Bell Aircraft Corporation, Buffalo, N. Y.

Nomenclature

R	=	distance from the center of the sun, in general
R_{equiv}	=	equivalent distance, i.e., the distance at which the local circular velocity (around sun) is equal to the equivalent circular velocity, defined by Equation [4]
R_P	=	distance at which perihelion velocity is attained
$R_\oplus$	=	distance of the earth
r	=	distance from the center of the earth, in general
$-r_\infty$	=	radius of the earth
r_s	=	distance of satellite orbit
r_p	=	distance at which local parabolic velocity is obtained
v_c	=	circular velocity with respect to the earth
v_p	=	parabolic velocity (escape velocity) with respect to the earth
v^*	=	any velocity with respect to the sun
$v_c{}^*$	=	circular velocity at distance $R_\oplus$
$v_{c,\,equiv}{}^*$	=	equivalent circular velocity around the sun, defined by Equation [4]
$v_p{}^*$	=	parabolic velocity (escape velocity from the solar system)
$v_P{}^*$	=	perihelion velocity in transfer ellipse to the target planet
♀	=	astronomical symbol of planet Venus
♂	=	astronomical symbol of planet Mars

IN A recent paper (1)[2] H. S. Tsien presented a very interesting discussion of the effect of acceleration on the energy requirements of space-bound vehicles, leaving a satellite orbit. Parabolic speed (escape velocity) was taken as the terminal speed of the vehicle and, for the more important case of tangential take-off (in contrast to radial take-off), the two boundary cases of very large and very small constant accelerations were treated.

This note presents some additional remarks on this subject. A space ship leaving for an interplanetary mission must attain hyperbolic speed rather than parabolic velocity $v_p = \sqrt{2g_s r_s}$ where g and r are gravitational constant and distance from the center of the earth, respectively, and the subscript s refers to the satellite orbit. The hyperbolic velocity, or start velocity is given by

$$V_{st} = \sqrt{v_p{}^2 + (\Delta v_{st})^2} \quad \ldots\ldots\ldots\ldots\ldots [1]$$

Received August 3, 1953.

[1] Development Engineer, Preliminary Design Department.

[2] Number in parentheses refers to Reference at end of paper.

where Δv_{st} is the rest velocity of the ship after escape. This velocity is the difference between local circular velocity around the sun, v_c^* (assuming circular orbits), and the perihelion velocity, v_P^*, required to reach the target planet

$$\Delta v_{st} = v_P^* - v_c^* \quad [2]$$

Using Tsien's two dimensionless parameters, χ and ν, where $\chi = (c/v_c) \log_e (m_0/m_1)$, c = effective exhaust velocity, $v_c = \sqrt{g_s r_s}$ = circular velocity in the satellite orbit, m_0/m_1 = mass ratio of the vehicle, and ν the vehicle's (constant) acceleration in terms of g, one obtains, in analogy with Tsien's result for parabolic velocity

$$\text{for } \nu \to \infty: \quad \chi \to \sqrt{2} + \frac{\Delta v_{st}}{v_c} - 1 \quad [3]$$

In the opposite boundary case, where the acceleration approaches zero, the vehicle first moves away from the earth, at constant speed, v_c, corresponding to its orbit of departure, until a distance r_p is reached at which this velocity equals the local parabolic velocity (Fig. 1). At this point the vehicle's energy level with respect to the sun is equal to that of the earth, the velocity being the orbital velocity of the earth, v_c^*. From now on the vehicle moves away from the earth's orbit in the same manner as it moved out of the satellite orbit before. Its velocity remains constant and equal to v_c^*, while the distance increases, until, at a distance R_P, the difference between v_c^* and the local circular velocity is equal to Δv_{st}.

The above described process can be simplified by referring to the sun only, introducing an equivalent circular velocity $v_{c,\,\text{equiv}}^*$ defined by

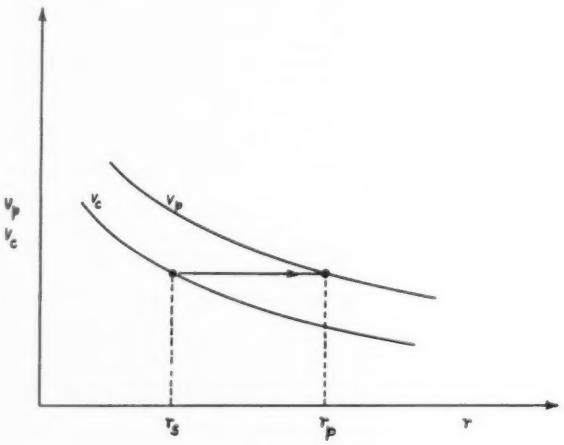


FIG. 1 ESCAPE WITH $\nu \to 0$

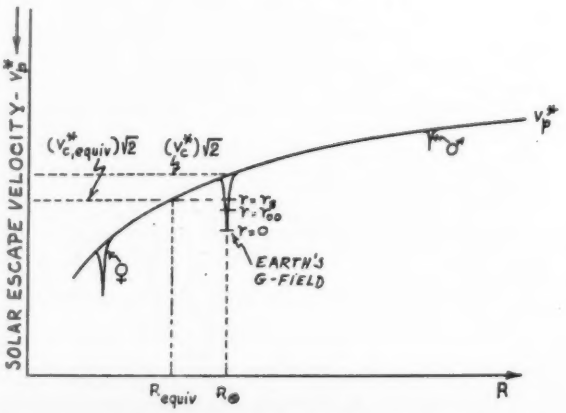


FIG. 2 EQUIVALENT CIRCULAR VELOCITY

$$v_{c,\,\text{equiv}}^* = \sqrt{(v_c^*)^2 + (v_p)^2 r_s} \quad [4]$$

and illustrated in Fig. 2. This velocity lies inside the earth's orbit at a distance of $R_{\text{equiv}} < R_\oplus$. Thus

$$\text{for } \nu \to 0: \quad \chi \to \sqrt{2 + \frac{(v_c^*)^2}{v_c^2}} \quad [5]$$

As increasingly great distances are selected for the initial satellite orbit, $v_p \to 0$ and, by introducing $\chi' = (v_c/v_c^*)\chi$

$$\chi' \to 1 \quad [6]$$

The process can then be presented in a graph similar to the one used for escape with $\nu \to 0$, as shown in Fig. 3; the case $v_p > 0$ holding for departure from inside the earth's gravitational field, $v_p = 0$ for departure from the earth's orbit in absence of the earth. The latter case has no practical significance.

In a paper which the author recently submitted for publica-

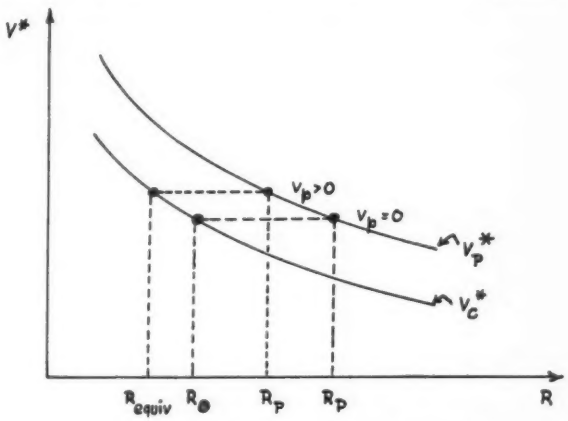


FIG. 3 ATTAINING PERIHELION SPEED WITH $\nu \to 0$

tion, the existence of an optimum satellite orbit, individually different for each target planet, has been demonstrated analytically. For the optimum satellite orbit, V_{st} (Equation [1]) becomes a minimum for the target planet in question. It was found that the distance $r_{s,\,\text{opt}}$ of this orbit is defined by the condition

$$\Delta v_{st} = (v_p)_{r_s,\,\text{opt}} = \sqrt{2 g_s r_{s,\,\text{opt}}} \quad [7]$$

so that

$$V_{st,\min} = (v_p)_{r_s,\,\text{opt}} \sqrt{2} \quad [8]$$

Then, for very high accelerations, the minimum value for χ becomes

$$\text{for } \nu \to \infty: \quad \chi_{\min} \to 2\sqrt{2} - 1 = 1.8284 \quad [9]$$

for any planet.

The conclusion is that for a $\nu \to 0$ vehicle (e.g., ion rocket), χ decreases monotonously with increasing distance of the orbit of departure (Equations [5] and [6]); for a $\nu \to \infty$, ($\nu > 0.50$ g) vehicle (e.g., thermochemical or thermonuclear rocket), there is only one satellite orbit for which χ becomes a minimum (Equation [9]).

For flights along minimum energy ellipses to the respective planets, these minima correspond to terrestrial satellite distances, in nautical miles, of 4670 for Mercury, 57,500 for Venus, 49,400 for Mars, and 5560 for Jupiter. They are flat, but involve reductions in V_{st} which cannot be neglected. A space ship, starting with a small constant acceleration of, say 0.25 or 0.15 g, will increase its radial distance considerably until terminal velocity is attained. If the orbit of departure is inside of such a minimum, as certainly will be the case for Mars and Venus, then the vehicle will move toward

optimum distance while under power. This effect will reduce to a certain degree the penalty connected with prolonged burning along an increasingly steep path, thereby reducing the losses connected with smaller acceleration.

From these considerations it appears that for the hyperbolic case no single curve of general validity, connecting the two boundary conditions $v \to 0$ and $v \to \infty$, can be obtained, as is possible for the parabolic case treated in Ref. (1).

Reference

1 "Take-Off from Satellite Orbit," by H. S. Tsien, JOURNAL OF THE AMERICAN ROCKET SOCIETY, vol. 23, July-August 1953, pp. 233–236.

Compressibility Studies of Rocket Propellants

K. R. STEHLING[1] and A. A. KOVITZ[2]

Bell Aircraft Corporation, Buffalo, N. Y.

IT was with considerable interest that the paper by G. G. Kretschmar, "The Velocity of Sound in Some Rocket Propellant Liquids" [ARS JOURNAL, vol. 23, March-April, 1953, p. 82], was read and his results compared with those of the authors, obtained at the Bell Aircraft Corporation. Work of a similar nature was completed at Bell Aircraft and reported under "Compressibility Measurements of Some Rocket Engine Propellants" [AF Contract no. W33-038ac 14169, Report no. 56-982-013, Sept. 15, 1952].

Liquids of common interest to both investigations were JP-3 and JP-4, hydrocarbon mixtures often used as jet fuels. While Kretschmar measured the velocity of sound in anhydrous hydrazine, the liquid studied at Bell Aircraft Corporation was eutectic hydrazine (68 per cent hydrazine in water). Measurements on white fuming nitric acid were not made, due to limitations in the apparatus.

The program at Bell Aircraft was initiated as part of a more general study of rocket combustion instability. One of the major physical constants which had to be determined was the compressibility of the various liquid propellants. The only feasible method of measuring the compressibility was through the "indirect" approach utilizing the velocity of sound and the formula $V = \sqrt{g/K\rho}$, where V = velocity of sound (ft/sec), ρ = density (lb/ft^3), g = acceleration due to gravity, and K = compressibility (ft^2/lb).

The authors also discarded the use of Kundt's tube and other standing wave apparatus after several inconclusive experiments. It was then decided to try ultrasonic interferometry.

The general form of the apparatus was very similar to that shown in Fig. 3 of the Kretschmar paper. However, a 1000-ml Pyrex beaker was used instead of the stainless-steel container shown in Fig. 3. As a source for the ultrasonic energy, a barium titanate disk, 1-in. diam. and $^1/_8$-in. thickness, was placed under a similarly shaped, stainless-steel reflector attached to a micrometer shaft.

The barium titanate polycrystal was driven by a Hewlett-Packard oscillator at approximately 128 kc, the resonance frequency of the crystal. The periodic half-wave-length positions along the standing wave pattern between the crystal and reflector were indicated by a Dumont 304-H oscilloscope, which measured the voltage across a balanced bridge network. The distance between two similar deflections on the CRO was taken as the half wave length of the sound wave.

A direct comparison of the results is of interest, with a tabulation of the compressibility as calculated at Bell Aircraft:

Liquid	Source	V, ft/sec	$K \times 10^{-4}$, ft^2/lb
JP-3	Bell Aircraft	4040 to 4060	4.11
	Kretschmar	Sample a: 4045	
		Sample b: 3921	
JP-4	Bell Aircraft	4050 to 4060	4.10
	Kretschmar	Sample a: 4073	
		Sample b: 4073	
Hydrazine (anhydrous)	Kretschmar	6751	...
Hydrazine (eutectic)	Bell Aircraft	7096	0.992

Received September 3, 1953.

[1] Former Unit Leader, Rocket Research Group; now Research Associate, Department of Aeronautical Engineering, Princeton University, Princeton, N. J. Member ARS.

[2] Former Rocket Research Engineer, Rocket Research Group; now Graduate Student, Department of Aeronautical Engineering, Princeton University, Princeton, N. J.

Kinetics of the Combustion Wave

(*Continued from page 354*)

14 "Burning Velocities of Acetylene and Dideuteroacetylene with Air," by R. Friedman and E. Burke, *Industrial and Engineering Chemistry*, vol. 43, 1951, pp. 2772–2776.

15 "Flame Propagation," by D. M. Simon, *Industrial and Engineering Chemistry*, vol. 43, 1951, pp. 2718–2721.

16 "Spectroscopy and Combustion Theory," by A. G. Gaydon, Chapman and Hall, London, 1948; and later papers by Gaydon and H. G. Wolfhard, *Proceedings of the Royal Society of London*, 1948–1952.

17 "On the One-Dimensional Theory of Flame Structure," by R. Friedman and E. Burke, *Journal of the Aeronautical Sciences*, vol. 18, 1951, pp. 239–246.

18 "Heat Capacity Lag in Gases," by R. Walker, *NACA Technical Note* 2537, 1951.

19 "Elementary Reactions in the Gas Phase Involving Excited Electronic States," by K. J. Laidler and K. E. Shuler, *Chemical Reviews*, vol. 48, 1951, pp. 153–224.

20 "Quantitative Studies of Apparent Rotational Temperatures of OH in Emission and Absorption," by S. S. Penner, *Journal of Chemical Physics*, vol. 21, 1953, pp. 31–41.

21 "Measurements of Normal Burning Velocities and Flame Temperatures of Bunsen Flames," by J. W. Andersen and R. S. Fein, idem., vol. 17, 1949, pp. 1268–1273.

22 "Temperature Profiles in Propane-Air Flame Fronts," by R. M. Fristrom, R. Prescott, R. Neumann, and W. H. Avery, Fourth Symposium on Combustion, Williams and Wilkins, Baltimore, 1953, pp. 267–274.

23 "A Method for the Measurement of the Temperature Distribution in the Inner Cone of a Bunsen Flame," by G. Dixon-Lewis and M. J. G. Wilson, *Transactions of the Faraday Society*, vol. 47, 1951, pp. 1106–1114.

24 "A Method of Analysis of a Plane Combustion Wave," by J. H. Burgoyne and F. Weinberg, Fourth Symposium on Combustion, Williams and Wilkins, Baltimore, 1953, pp. 294–302.

25 "Measurements in the Reaction Zone of a Bunsen Flame," by H. Klaukens and H. G. Wolfhard, *Proceedings of the Royal Society of London*, vol. 193(A), 1948, pp. 512–524.

26 "Measurement of the Temperature Profile in a Laminar Flame," by R. Friedman, Fourth Symposium on Combustion, Williams and Wilkins, Baltimore, 1953, pp. 259–263.

27 "Experimental and Theoretical Studies of Flame-Front Stability," by G. H. Markstein, *Journal of the Aeronautical Sciences*, vol. 18, 1951, pp. 199–209.

28 "Nonisotropic Propagation of Combustion Waves in Explosive Gas Mixtures and the Development of Cellular Flames," by J. Manton, B. Lewis, and G. von Elbe, *Journal of Chemical Physics*, vol. 20, 1952, pp. 153–157.

29 "A Possible Relation Between Cellular Flame Structure and Quenching," by R. Friedman, *Journal of the Aeronautical Sciences*, vol. 18, 1951, p. 499.

30 "The Wall-Quenching of Laminar Propane Flames as a Function of Pressure, Temperature, and Air-Fuel Ratio," by R. Friedman and W. C. Johnston, *Journal of Applied Physics*, vol. 21, 1950, pp. 791–795.

31 Fourth Symposium on Combustion, Williams and Wilkins, Baltimore, 1953.

32 "Combustion, Flames and Explosion of Gases," by B. Lewis and G. von Elbe, Academic Press, New York, 1951.

33 "Flames; Their Structure, Radiation, and Temperature," by A. G. Gaydon and H. G. Wolfhard, Chapman and Hall, London, 1953.

Jet Propulsion News

ALFRED J. ZAEHRINGER, Thiokol Corporation, *Acting Editor*

JOSEPH C. HOFFMAN, General Electric Corporation, *Contributor*

Rockets and Guided Missiles

A NEW re-usable liquid-propellant JATO has been developed by the Aerojet-General Corporation. The nozzle is made of Niafrak (trade name for a silicon carbide made by the Carborundum Company) bonded with silicon nitride for thermal shock resistance.

A NEW model of the DeHavilland ATO unit, the "Super Sprite" (Fig. 1) has been announced. Oxidizer is hydrogen peroxide; fuel is kerosene or gasoline. A solid catalyst system is employed. Details: maximum thrust, 4000 lb; total impulse, 150,000 lb sec; operating time, about 40 sec; weight of empty unit is 600 lb.

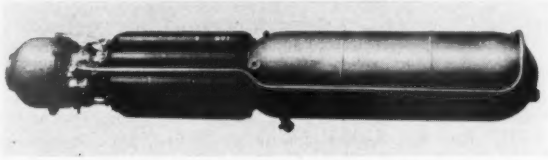


FIG. 1 THE DE HAVILLAND SUPER SPRITE ROCKET ENGINE

GUIDED missiles are not only being launched from the Air Force Missile Test Center at Cape Canaveral, Fla., but landed there as well, according to recent news releases. After the AFMTC-launched missiles have passed over the Grand Bahamas some 200 miles southeast, they are turned around 180 deg and returned to the Cape where they are brought to the ground as gently as possible. In the past some of the missiles were allowed to fall into the sea or were destroyed. In the future, according to the Air Force, an island chain of nine control points will be established and rockets can then be directed over a 1000-mile course from AFMTC to Puerto Rico and return.

THE use of rocket motors for wind-tunnel power has been suggested by scientists of the University of California. Six 5000-lb thrust rocket motors would furnish the primary jet for a tunnel having a 4 ft × 6 ft test section, a 400-mph speed, and an altitude capability from 5000 to 60,000 ft. The main advantage of such a scheme is the relatively low cost. About $250,000 could provide this tunnel as compared to the $1 million-plus needed for the more conventional tunnels. Operating cost would range from about $400 to $150 per operating minute. It was assumed that some standard rocket motor could be used. Other advantages for the system are its greater flexibility and its easily modified air speed.

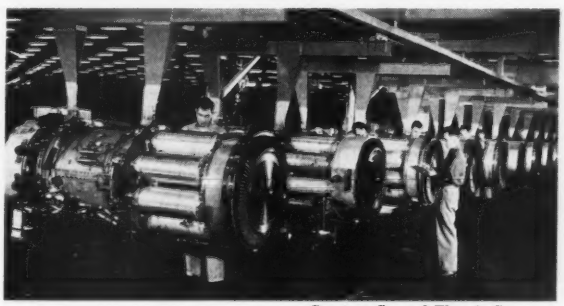
DUNCAN SANDYS, British Minister of Supply, recently stated that Britain had developed rocket missiles with speeds of 2000 mph which are guided by radar beams. It was claimed that such missiles could outmaneuver any piloted aircraft. Although no mention was made of other characteristics, it was hinted that a multistage, ground-to-air antiaircraft rocket was in the offing. Guidance is by a beam rider to a self-contained homing device.

A NEW British guided missile has its rocket booster unit mounted on the forward portion of the missile body in contrast to current American practice which favors a tail mounting. A cluster of eight rocket boosters in four packages blast their exhausts through the gaps between the missile wings. At separation, the boosters pull away radially. This mountng and separation scheme was used on the German Schmetterling (Butterfly) missile developed during the late stages of World War II.

LIQUID-propellant rockets should mean a tremendous cost reduction in operating rocket sleds, according to Robert King, civilian head of the Track Branch at Edwards AFB. King claims that liquid propellants are just coming into use now for the sled application and that three liquid-propellant rocket-sled power plants will soon be in operation at the Edwards 10,000-ft track. Fuel savings should be great, King says, because the cost of liquid propellants for a typical sled run is about $^1/_{25}$ that of solid propellants for the same sled speed. The rocket sleds at Edwards and Holloman AFB are used for a variety of research tasks including aeromedical and aerodynamic studies, and acceleration and deceleration tests on high-speed aircraft and guided missile components. Speeds close to 1000 mph are possible with these sleds.

Turbojet Engines

FIG. 2 shows new J-47 jet engines coming down the production line at General Electric Company's plant in Schenectady. According to General Electric, it is the first completely mechanized assembly line for jet engines in the country. The "grape arbor" system permits automatic movement from station to station and has resulted in a production increase.



Courtesy General Electric Company

FIG. 2 PRODUCTION LINE OF G-E J-47 JET ENGINES

AS A result of the experience gained during the first 13 months of the BOAC Comet services, the Ghost engines operated by both BOAC and the French Company, Union Aéromaritime de Transport, have officially been cleared to operate for 600 hours between overhauls. At Comet speed this is equivalent to flying ten times around the world, or the distance from the earth to the moon. Following the official

EDITOR'S NOTE: The information reported in this Section has been selected from approved news releases originating with the Department of Defense, private manufacturers, universities, etc., and from published news accounts in journals and newspapers. The reports are considered generally reliable, although no attempt has been made to verify them in detail.

INDUSTRIAL SOUND CONTROL ● INDUSTRIAL SOUND CONTROL ● INDUSTRIAL SOUND CONTROL ● INDUSTRIAL SOUND CONTROL ● INDUSTRIAL SOUND CONTROL ● INDUSTRIAL SOUND CONTROL ● INDUSTRIAL SOUND CONTROL ●

rocket ROAR silenced by ISC mufflers

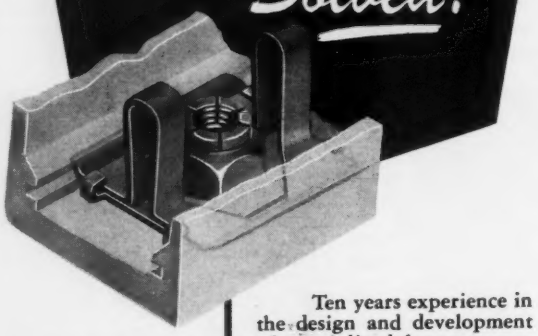
● In the laboratory, in the test cell, on the airfield, hundreds of INDUSTRIAL SOUND CONTROL installations are subduing the noise, heat and gas velocities generated during testing of the big jets.
Whatever *your* noise problem, ISC's skilled engineering, design, and installation "know how" — gained through years of practical experience — can help you, and is at your instant service.
We welcome the challenge of the unusual problem. For full information

WRITE ISC TODAY

Foreign Licensees:
Cementation (Muffelite) Limited, London
Les Travaux Souterrains, Paris

Industrial Sound Control, Inc.
45 Granby Street, Hartford, Conn.
2119 So. Sepulveda Blvd., Los Angeles, Calif.

FASTENING PROBLEMS Solved!



This floating anchor nut solved a costly problem. Formerly, 3 separate parts made installation difficult and costly; removal was impossible. Our design, easily installed and easily removed, cut costs sharply.

Ten years experience in the design and development of *specialized* fasteners, enables us to help you. Consultation on your problem, especially in the *design* stage, will result in a cheaper, more efficient fastener for your product. *Write today.*

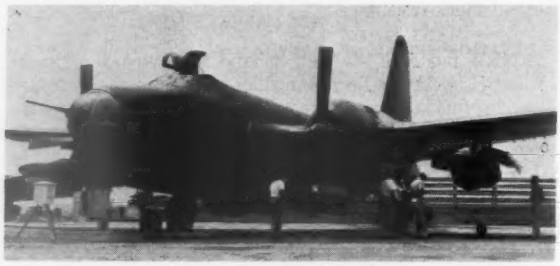
RESEARCH DIVISION
Nutt-Shel
811 AIRWAY
GLENDALE 1, CALIF.
Manufacturers of
SELF-LOCKING ANCHOR NUT
BOLT AND NUT RETAINERS

approval for a 600-hr life, a batch of BOAC engines immediately commenced running a trial extension to 750 hr.

Aircraft

AN INTERESTING feature of recent Regulus pilotless missile tests is that it is landed after the tests. A retractable landing gear is used in conjunction with a braking parachute.

HIGHER speed and greater safety are now possible with the Navy's big P2V patrol plane. Lockheed Aircraft Corporation announced the flight of the long-range craft using new J-34 jet engines to supplement its turbo-compound propeller engines. The P2V Neptune holds a nonstop record of 11,236 miles without refuelling (Fig. 3).



Courtesy Lockheed Aircraft

FIG. 3 NEPTUNE SPROUTS JET ENGINES

THE first test flights of another Navy Fury jet fighter, the FJ-3 produced by North American's Columbus plant, have been completed. The sweptwing, carrier-based craft is rated in the high subsonic class and is powered by a Wright J-65 turbojet engine. Wings and tail are swept 35 deg. Air intake for the engine is located in the nose. Claims are for a greater speed and increased climb rate over the previous FJ series.

THE greatest altitude record reached by man is the 83,235-ft altitude achieved by Lt. Col. Marion Carl, USMC, in the Douglas D-558-II Skyrocket recently. Col. Carl's 15.9 mile record topped the previous mark set in the same plane about two years ago. In this flight, the rocket plane was born aloft in the belly of a B-29 to 33,500 ft. After dropping down to 28,000 ft (due in part, it is claimed, to a faulty rocket chamber), the plane was accelerated in a 40-deg climb to the point of fuel burnout at 75,000 ft. From there, the Douglas research craft "coasted" up to its 83,235-ft mark. During the flight, the marine pilot was wearing a new full-pressure suit for protection in the event of failure of the cockpit pressurizing mechanism. The new suit, developed for BuAer by the David Clark Company of Worcester, Mass., was designed for altitudes above 50,000 ft.

ANOTHER Douglas Aircraft Company research plane recently made news when it was announced that the X-3 had been flight-tested at Mach 1.25. Designed from the outset for supersonic speeds, the X-3 is powered by two Westinghouse J-34 turbojet engines with afterburners. Although already in the supersonic range, it is said that the X-3 is underpowered, at present, to reach its design speed of Mach 3.

THE Navy Bureau of Aeronautics announced that Chance Vought Aircraft Division of United Aircraft Corporation had won a design competition for a new day fighter. Although details of the new jet fighter have been withheld, it was stated that it would have higher performance characteristics than present models. Chance Vought is currently producing the F7U-3 Cutlass fighter for the Navy and the guided missile Regulus. The company also received a contract for the A2U-1, an attack version of the Cutlass. The Cutlass (Fig. 4) is a carrier-based, twin-jet fighter. Now in quantity pro-

Courtesy United Aircraft Corporation

FIG. 4 THE U. S. NAVY CUTLASS, A CARRIER-BASED, TWIN-JET FIGHTER

duction, the Cutlass has been described as having exceptional operating characteristics. Top speed is cloaked in security regulations. The nose boom on the airplane shown in the photograph houses test instruments.

RECENT Air Force procurement orders have identified, officially, the power plants for some of America's newest jet aircraft. The North American F-100 (Super Sabre Jet), the McDonnell F-101, and the Convair delta-winged F-102 all will be powered by P&W J-57 turbojet engines with afterburners. The Republic F-105 and the Douglas B-66 will have Allison J-71 turbojet engines.

ONE hour and 26 minutes of flight at supersonic speeds is the claim for a North American F-86A Sabre Jet. The ship is a specially equipped, high-altitude, supersonic flying laboratory of the NACA and operates from the Ames Aeronautical Laboratory at Moffett Field, Calif. The plane features special rings, test fittings, interchangeable parts, and, at times, a radical and unusual configuration to enable NACA's scientists to obtain and evaluate new information on flight above the Mach 1 level. Best information available indicates that this plane has had more flights above Mach 1 than any vehicle on record.

ANOTHER NACA research plane at Ames, a modified North American YF-93, incorporates an unusual device for the measurement of thrust during flight. According to a recent NACA announcement, the device consists of a survey probe that is moved rapidly pendulum-fashion across the tailpipe outlet, automatically recording thrust readings as it passes through the jet stream. This YF-93 is also equipped with four RATO units to enable it to reach its high operating speed quickly.

ACCORDING to North American Aviation, the F-100 has made supersonic flights in level flight without the benefit of an initial dive. Powerful 15,000-lb thrust turbojet engine makes the F-100 America's first truly supersonic tactical fighter.

THE first Avon-powered F-86 Sabre, produced by Commonwealth Aircraft Corporation, Auburn, Australia, was successfully test-flown. To accommodate the larger engine, the forward fuselage was enlarged.

THE PTL-022 turboprop engine is said to power two models of Soviet long-range bombers. The TuG-75 ("America" bomber) is powered by six of these engines, while a Type 31 (a modified Tu-4) uses four of the engines; the 31 is reported as having a speed of 450 mph and a range of 6500 miles. The turboprop power plant is said to be a modified German Junkers engine. Horsepower of the engine is about 5000 ehp with a

dyna-gage

an electronic instrument for the measurement of static and dynamic pressures

standard instrument of the rocket industry . . . for indicating combustion pressures

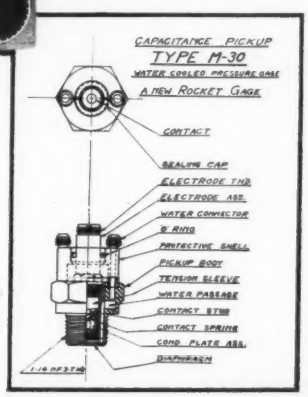
DYNA-GAGE DG-101

Output: ± 10 volts at 25,000 ohms.

Power requirements: PS-102.

Size: 8⅜" x 11½" x 11½"

Versatile, positive, dependable — under the most adverse conditions



■ The water-cooled capacitance pick-up permits the measurement of pressures at extremely high combustion temperatures. Remote location of the pressure indicator is provided by cable connection, with lengths up to 100 feet performing satisfactorily. The pressure indicator can be installed with the diaphragm flush with the wall of the pressure chamber, eliminating surge effects.

Write or telephone for information

PHOTOCON RESEARCH PRODUCTS

421 N. Foothill Blvd., Pasadena 8, Calif. • Phone SYcamore 2-4131

kerosene fuel consumption of about 750 gal per hr. Design static thrust rating is 11,500 lb. Other details of the PTL-022 are: weight, 6400 lb; diameter, 5 ft, 4 in.; length, 20 ft, 5 in.

THE world's most heavily armed fighter is the claim for the Canadian CF-100. This twin-jet, the latest addition to the RCAF, carries 120 electronically aimed and fired solid propellant rockets in its wing-tip pods. In production at Avro-Canada, the plane is powered by two Orenda turbojet engines which give the plane near supersonic speed capabilities.

Dayton Aircraft Show

ALTHOUGH jetsand helicopters shared top positions at the National Aircraft Show, September 5–7, 1953, in Dayton, Ohio, a few items of particular interest to rocket men were unveiled for the first time.

The all-rocket powered Bell X-1B supersonic research plane was placed on public view for the first time at the show. Flown to Dayton in the belly of a B-29, the X-1B model on display was incomplete and was scheduled for return to Bell prior to its flight tests which are due shortly at Edwards AFB. The X-1B is about five feet longer than the X-1 and is powered by an RMI rocket engine. The powered flight time of the unit is about 4 minutes; operating speed should be close to 1650 mph.

Smokeless rocket-assisted take-off units developed for the Navy by Aerojet were flight tested on a Lockheed P2V Neptune at the aircraft show. Observers reported that only a trace of smoke was generated by the Navy RATO units. A new, streamlined chamber is used in the unit which is officially designated as the 15KS-1000 RATO.

RATO units will be used on Boeing's six-jet training version of their B-47 Stratojet. Shown to the public for the first time, the plane (designated as the ETB-47B) had RATO ports in the fuselage sides behind the wings.

Another rocket innovation unveiled was a new rocket rack built by Oerlikon. Mounted on a Fletcher FD-25B (tactical support) wing section, the rack can take 5 or 8-cm rocket projectiles.

Facilities

LOCKHEED Aircraft Corporation of Burbank, Calif., has begun construction of an icing wind tunnel to test parts of high-performance jet models now under development. The 9600-cu ft icing tunnel is capable of cooling down to minus 50 deg. Among parts scheduled for test are airplane wings, control surfaces, deicer boots, engine heater air inlets, carburetor intakes, and built-in antennas and radomes.

THE Navy is to build a jet engine testing plant at Perry, Ohio. The $1,286,000 facility is to be operated by Thompson Products of Cleveland for the evaluation of materials for new aircraft gas turbines.

HANDLEY Page Ltd. of Radlett, England, has placed a high-subsonic wind tunnel into operation. A novel feature of the tunnel is that Rolls-Royce "Nene" turbojet engines of 5000-lb thrust provide the drive. Three of these engines are staggered vertically in the tunnel and are used to induce air flow through the working section which measures 4 × 3 ft in cross section by 8 ft long.

A LABORATORY building costing $1.4 million is to be built for Battelle Memorial Institute of Columbus, Ohio. Added space will be for projects with government and industry in chemical engineering research.

THE "White Sands of the North" is the term recent news releases have applied to Canada's new rocket-testing range at Cold Lake, northeast of Edmonton, Alberta. Costing about $35 million, the range should be ready for use early in 1954. The project is expected to cover about 4000 sq mi and have 45 all-weather structures. It is thought that upward of 2000 technicians and their families will live year-around in a completely self-contained settlement.

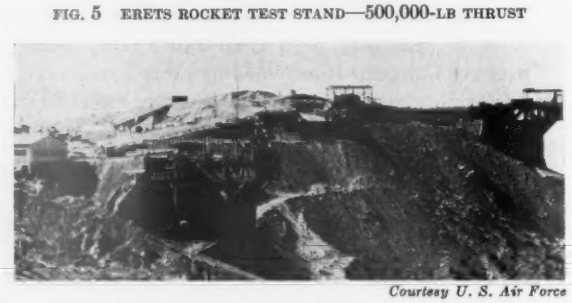
Air Force Experimental Rocket Engine Test Station

THE Air Force has recently revealed that it is operating giant rocket test stands at its Experimental Rocket Engine Test Station at Edwards AFB, Calif. As the thrust ratings of rocket engines increased, it soon became impractical and uneconomical to test rocket missiles in free flight. The need for a facility capable of static testing missile propulsion systems, both the individual components and in entirety, became imperative. In 1951, the ERETS was established in the Mojave Desert about 31 miles from Edwards AFB.

To handle new rockets and missiles, thrust capacities up to half a million pounds can be accommodated in stands such as the one shown in Fig. 5, which is over 100 ft high and has concrete footings penetrating as deep as 60 ft into solid granite. Fig. 6 shows a view of the technical area of ERETS with the central control station located midway between two stands. Fig. 7 shows how various missiles and engines are tested. Fig. 8 shows the interior of the central control room. At the left the engineer in charge stands at the main control console. In the center an engineer uses a periscope to observe the firings, while at the right a technician checks the banks of automatic recorders.

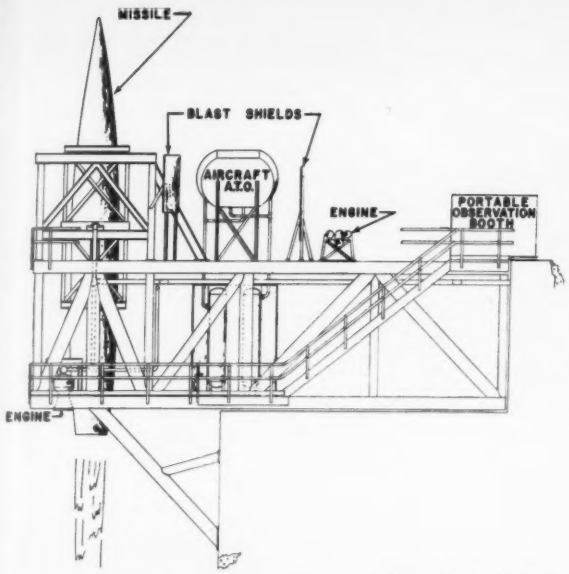
Courtesy U. S. Air Force

FIG. 5 ERETS ROCKET TEST STAND—500,000-LB THRUST



Courtesy U. S. Air Force

FIG. 6 ERETS TEST STANDS ON EITHER SIDE OF CONTROL STATION



Courtesy U. S. Air Force

FIG. 7 UTILIZATION OF TEST STAND AT ERETS

Courtesy U. S. Air Force

FIG. 8 INTERIOR OF CENTRAL CONTROL ROOM AT ERETS

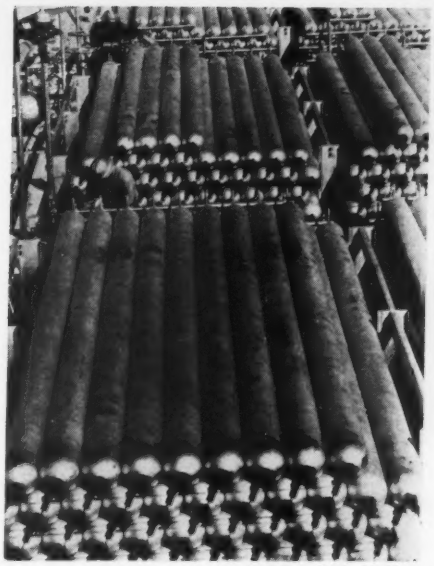
ERETS is nearly self-sustaining and incorporates office and work space, maintenance and general machine shops, transportation facilities, photographic services, and housing and eating facilities for transient personnel. One of the practical features of the station is the use of intermediate storage of thousands of gallons of propellants such as nitric acid and liquid oxygen. Fig. 9 shows the multiple banks of gaseous nitrogen under high-pressure which is used to operate the fuel systems in the rocket tests.

Two of the missiles which have been tested here are the F-99 Bomarc (Boeing) and Navajo (North American). Other test details are still classified.

Rocket Monofuel Conference

ON SEPTEMBER 22 and 23, forty-seven rocket experts from the United States and Great Britain gathered at the Wyandotte Chemicals Corporation Research Center to discuss the most recent developments in rocket monopropellants. This conference was sponsored jointly by the U. S. Navy Bureau of Aeronautics and Wyandotte Chemicals Corporation.

Intense interest centers in these unique fuels which require



Courtesy U. S. Air Force

FIG. 9 COMPRESSED NITROGEN SUPPLY FOR FUEL SYSTEM OPERATION, ERETS

no source of oxygen to deliver their thrust in a rocket motor, or power a high-pressure gas turbine. Whereas standard rocket propellants require both a fuel and oxidizer, a monopropellant functions alone in an entirely different manner, greatly simplifying problems of fuel storage, handling, pumping, and ignition.

When the Germans launched their V-2 missiles from France onto London in World War II, hydrogen peroxide was used as a monofuel to pump the alcohol and liquid oxygen was used to power the rocket motor. Since then, extensive research, largely in the United States and Great Britain, has led to several monopropellants useful for many diverse applications. Thus, these monofuels could find their place in jet-assisted-takeoff (JATO) units, jet pumps and centrifugal blowers in applications to aircraft, catapult power plants for carrier and land-based aircraft, and ducted rockets which use the rocket motors for take-off and then use ram air for combustion when in flight.

As originator of one of today's important monofuels, Wyandotte Chemicals Corporation was selected as host for this Rocket Monofuel Conference.

The general chairman of the Conference was Ben F. Coffman, Head of the Rocket Branch of the Bureau of Aeronautics. The conferees were welcomed to Wyandotte by Dr. Thomas H. Vaughn, Vice-President—Research and Development, Wyandotte Chemicals Corporation.

Moderators for the five sessions were Dr. H. F. Calcote of Experiment, Inc., Richmond, Va., Mr. Charles H. King, United Aircraft Corporation, East Hartford, Conn., Mr. David A. Young, Aerojet-General Corporation, Azusa, Calif., Mr. Al Thatcher, Reaction Motors, Inc., Rockaway, N. J., and Dr. Arthur B. Ash, Wyandotte Chemicals Corporation.

The Arnold Engineering Development Center

THE Arnold Engineering Development Center (AEDC) was established by the U. S. Air Force to provide facilities for the evaluation and testing of supersonic aircraft, aircraft engines of all types, and guided missiles under simulated free-flight conditions. The facility was named in honor of the late General H. H. Arnold who recognized the need for the expansion of our air research and development facilities and who, in 1944, asked Dr. Theodore von Kármán to make a

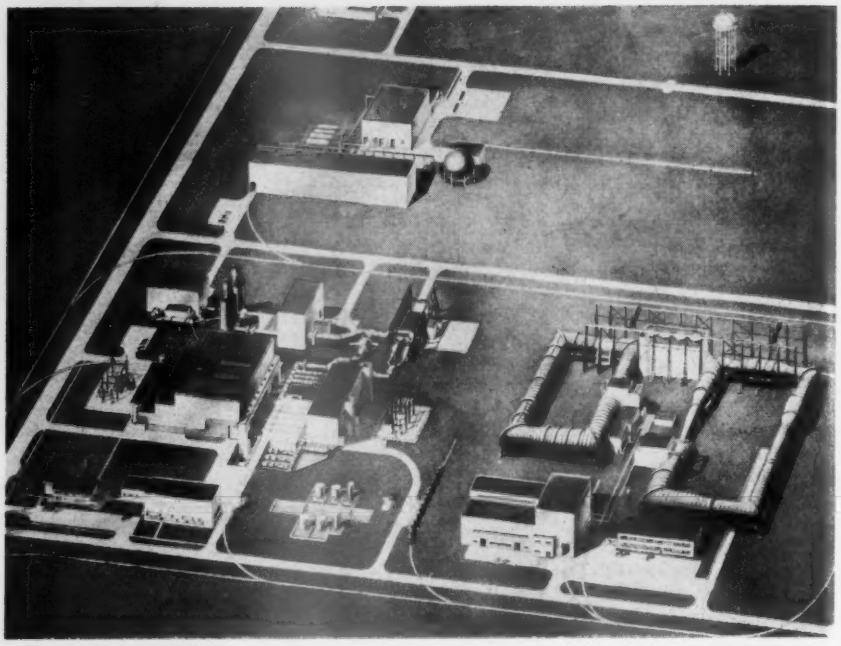
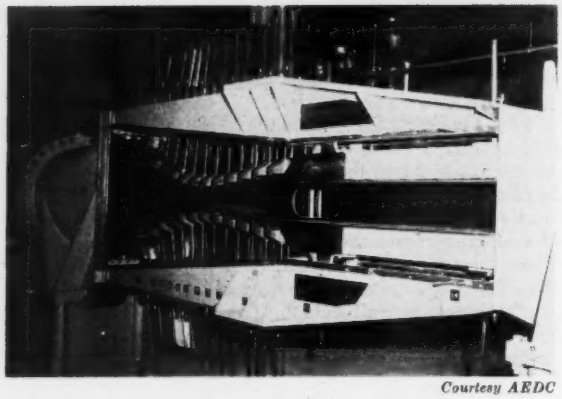


FIG. 10
ARTIST'S CONCEPTION OF THREE MAJOR TEST FACILITIES UNDER CONSTRUCTION AT THE AEDC, TULLAHOMA, TENN.
Courtesy AEDC



Courtesy AEDC

FIG. 11 GAS DYNAMIC FACILITY (TUNNEL E-1) AT THE AEDC

study of air development facilities. The 81st Congress authorized construction of AEDC in 1949. AEDC is a major Air Force facility, but it has been announced that its testing capacity will be made available to other military services, industrial and educational institutions engaged in aerodynamic, propulsion research and development. The AEDC facilities are operated by Aro, Inc., under contract with the U. S. Air Force.

AEDC is located near Tullahoma, Tenn., about 85 miles northwest of Chattanooga. The location was chosen because of the availability of TVA electric power (the power demand is equivalent to that of a city of over 100,000 population) and water (at maximum flow the cooler for the propulsion wind tunnel will use water at a rate equivalent to a city the size of Washington, D. C.).

Fig. 10 shows an artist's conception of the three major test facilities presently under construction at AEDC. In the lower left corner is the Engine Test Facility, the first of the three major test facilities to be completed. It will have three test chambers and a test bed for research, development, and evaluation programs on turbojet, turboprop, and small ramjet engines under simulated flight conditions. Altitudes up to about 80,000 ft and temperatures as low as −120 F can be reached. The Gas Dynamics Facility can be seen in the upper left portion of the print. It will consist of a group of supersonic wind tunnels to be used for the development testing of models of aircraft, projectiles, and missiles designed to operate at extremely high speeds; some of these tunnels will operate in the hypersonic range. Fig. 11 shows a picture of Tunnel E-1 with variable nozzle geometry. In the lower right is the Propulsion Wind Tunnel which will consist of two large tunnels, one transonic and one supersonic, each having 16-ft-square test sections.

High-Altitude Research

AS MAN flies higher and higher, data of the upper atmosphere become increasingly important. Operation "Moby Dick" has been initiated by the ARDC of the Air Force to learn more about the 50,000–100,000 ft levels. Balloons are utilized in the project. Sizes vary from 50 to 150 ft in diameter. The polyethylene bags can carry payloads up to 500 lb and can stay aloft for three days.

A REMOTE-controlled rocket launching from an unmanned balloon was shown to be feasible during recent tests on Project Muskrat for the Office of Naval Research. The rocket, called Deacon, was carried aloft by a balloon to about 70,000 ft and then fired upward. During one test, the rocket reached an altitude of 50 miles. All flight tests on this project were made from the deck of the Navy Icebreaker, USS *Staten Island*, in the vicinity of the north geomagnetic pole. The rocket carried Geiger counters and ionization chambers for high-altitude studies of cosmic radiation and atmospheric pressure.

Safety and Jet Propulsion

ACCORDING to Navy scientists, high-powered radar can be extremely dangerous to personnel who walk in front of such sets while they are operating. Only recently have high-power outputs been available, and researchers have found that these can kill animals quickly. The most serious aspect is the possible production of eye cataracts, following exposure to microwave radiation. The Navy thinks that damage to the eye and hollow organs occur when subjected to radiations of 10-cm wave length. Although radiation danger from "low power" (45,000 watt) radar beams is not too serious, the Navy

believes the problem will be most vital with the 1,000,000-watt sets of tomorrow.

AS RESEARCH planes fly higher and faster, the Navy believes some system will be needed to warn pilots when they are nearing unconsciousness. A new telemetry system, which transmits back to scientists on the ground the details as to how a flier's mental and physical faculties are working, has just been disclosed by the Navy. Tiny electrodes fastened to numerous spots on the pilot's head and body pick up electrical impulses given off by the brain and muscles. These impulses are amplified 60,000 times and radioed back to the ground. There, flight surgeons can spot approaching danger and warn the pilot. This problem, too, will become more acute, according to the Navy, as research planes become operational and are flown by pilots having, perhaps, less physical stamina than is demanded of test pilots.

THE Navy is concerned over the effects of high-intensity noise generated by present jet engines and future guided missile ramjet and rocket power plants on humans. To find out more about these effects, a team of research personnel accompanied the aircraft carrier *Coral Sea* during a part of its 1952 operations. The data obtained during the trip re-emphasized the seriousness of the problem and, according to the Navy BuMed, dispelled any hopes that a quick and easy solution might be found.

At 50 ft from a jet-propelled F4U during open throttle, the noise intensity was rated at 135 decibels. This compares with the 50 decibels of ordinary conversation and 90 decibels for an auto horn. In general, an increase of three decibels in noise rating doubles the intensity of the noise.

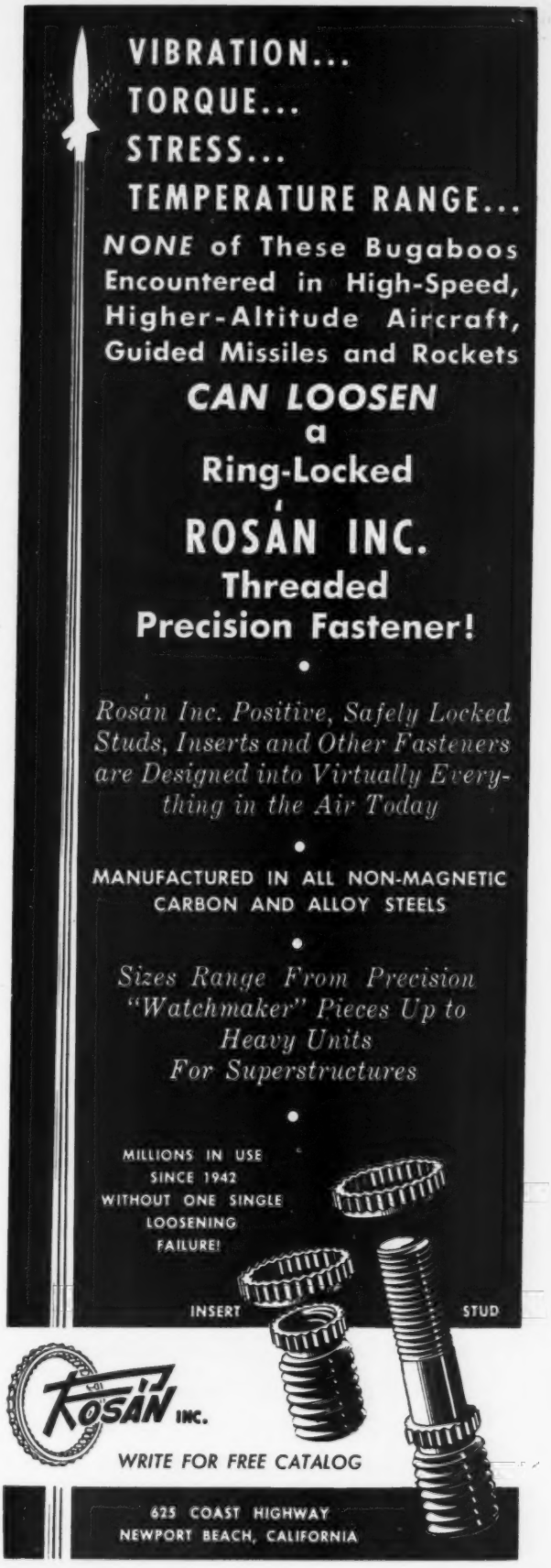
According to BuMed, noise has an extra-auditory effect about which little is known. However, it is known that intense stimulation of the nervous system may result in a temporary weakness and/or collapse of the individual exposed. Heating of the human body may occur in intense noise fields also.

Reducing the noise at its source is one remedy, but little is supposedly known about the generation of noise by power plants. Noise barriers (similar to blast deflectors) were not effective. One attractive solution is the use of ear-protective devices. The Bureau of Medicine and Surgery reports that a number of protective devices are under accelerated development and that a flight-deck crewman's helmet incorporating these devices will soon be in production.

Instrumentation and Materials

CONSOLIDATED Engineering Corporation, Pasadena, Calif., has introduced a vibration-measuring pickup measuring only 1 in. by 1 in. and weighing only 1.3 ounces. Its use is for testing aircraft turbines, rocket motors, and it is designed for continuous operation at 300 F, or for intermittent operation up to 100 hr at 500 F.

A COMPACT electronic mass flowmeter, weighing less than 8 lb and capable of measuring actual pounds of fuel consumption, has been announced by Gavco Corporation, a subsidiary of the General Aviation Corporation, New York, N. Y. The flowmeter system consists of a fuel-line sensing unit that measures the flow in gallons. Samples of fuel passing through the line are simultaneously weighed by a density detector. Information from the sensing unit and the density detector is continuously fed to an electronic integrator that compensates the flow for the true weight of the fluid and sends pulses to a dial indicator showing consumption in lb per hr. The flowmeter is said to have important applications in jet engine and rocket development.


VIBRATION...
TORQUE...
STRESS...
TEMPERATURE RANGE...
NONE of These Bugaboos Encountered in High-Speed, Higher-Altitude Aircraft, Guided Missiles and Rockets
CAN LOOSEN a Ring-Locked
ROSAN INC.
Threaded Precision Fastener!
Rosan Inc. Positive, Safely Locked Studs, Inserts and Other Fasteners are Designed into Virtually Everything in the Air Today
MANUFACTURED IN ALL NON-MAGNETIC CARBON AND ALLOY STEELS
Sizes Range From Precision "Watchmaker" Pieces Up to Heavy Units For Superstructures
MILLIONS IN USE SINCE 1942 WITHOUT ONE SINGLE LOOSENING FAILURE!
INSERT
STUD
ROSAN INC.
WRITE FOR FREE CATALOG
625 COAST HIGHWAY
NEWPORT BEACH, CALIFORNIA

CURRAN ENGINEERING CO.

Manufacturer of
MECHANICAL COMPONENTS
from
METALS, CERAMICS, AND PHENOLICS

Consultants and Specialists of
ROCKET IGNITER ASSEMBLIES
and
LONGITUDINAL SHAPED CHARGE CUTTERS

"CENGO" Process for
HIGH TEMPERATURE-HIGH DIELECTRIC INSULATING OF METALLIC ASSEMBLIES

4423 W. Jefferson Blvd.
Los Angeles 16 California.

INSTRUMENTATION SPECIALIST

to be in charge of electronic instrumentation in connection with research and development of liquid propellants. Permanent salaried position with expanding group in the research department of a major chemical company located in the Northeast. Requirements include: 2 years of college plus 3 years of experience or equivalent. Experience with electronic circuitry, high frequency transient pressure measurement, and some knowledge of liquid propellants or chemistry desired. Send complete résumé to Box 717A, % Journal of the American Rocket Society.

Courtesy B. F. Goodrich Company

FIG. 12 PYROLOCK-TREATED PANEL RESISTS FLAME

A NEW thermal insulating material called "Pyrolock," developed by B. F. Goodrich, protects metal for as long as 10 sec against flame temperatures hotter than the melting point of the metals. Fig. 12 shows a test. The untreated steel panel at right burned through in less than a second under an oxy-acetylene flame at about 5000 F. As this photograph was taken, the treated steel panel at left was unharmed after $4^1/_2$-sec exposure to the same flame.

ANOTHER new mass flowmeter system for accurately measuring the fuel consumption rate of jet aircraft has been announced by the General Electric Company's Meter and Instrument Department. The flowmeter measures the true mass rate of fuel consumption per engine in lb per hr and eliminates the density errors due to type of fuel and its temperature. Total weight of the system is about 11 lb.

A NEW technique for the stabilization of zirconium dioxide has been announced by ceramic research scientists at Wright Air Development Center, Dayton, Ohio. Special additives and new fabrication methods permit low costs and allow the material, which can withstand temperatures of over 4000 F, to be used in rockets and turbojets.

Minimal Rocket Trajectories

(*Continued from page 367*)

References

1 "The Determination of Minimal Orbits," by D. F. Lawden, *Journal of the British Interplanetary Society*, vol. 11, 1952, pp. 216–224.

2 "Orbital Transfer via Tangential Ellipses," by D. F. Lawden, *Journal of the British Interplanetary Society*, vol. 11, 1952, pp. 278–289.

3 "Inter-Orbital Transfer of a Rocket," by D. F. Lawden, presented at the Third International Congress on Astronautics, *Annual Report of the British Interplanetary Society*, 1951–1952, pp. 321–333.

4 "Optimum Thrust Programming for a Sounding Rocket," by H. S. Tsien and R. C. Evans, JOURNAL OF THE AMERICAN ROCKET SOCIETY, vol. 21, 1951, pp. 99–107.

5 "Optimum Burning Program for Horizontal Flight," by A. R. Hibbs, JOURNAL OF THE AMERICAN ROCKET SOCIETY, vol. 22, 1952, pp. 204–212.

6 "Minimal Trajectories," by D. F. Lawden, *Journal of the British Interplanetary Society*, vol. 9, 1950, pp. 179–186.

7 "Calculus of Variations," by A. R. Forsyth, Cambridge University Press, New York, N. Y., 1927, 656 pp.

American Rocket Society News

H. K. WILGUS, *Associate Editor*

Record Attendance Foreseen at ARS Eighth Annual Convention

McAlpin Hotel, New York, N. Y., December 2–4

THE Eighth Annual Convention of the AMERICAN ROCKET SOCIETY will take place on December 2–4, 1953, at the McAlpin Hotel, New York, N. Y., in conjunction with the ASME Annual Meeting which is being held concurrently at the nearby Statler and Governor Clinton Hotels.

This year's Convention, designed to cover the interests of a fast-growing membership, promises to attract a record-breaking attendance. As a round-up of new ideas, new developments, and new applications in the rocket and jet propulsion field, the program is the most comprehensive one ever presented by the ARS. Rocket and turbine testing, rocket motor design, thermodynamics, heat transfer, and combustion studies are some of the subjects to be discussed at the seven technical sessions at which 21 papers will be presented.

In addition to the technical sessions, the program includes a special tour of the Hayden Planetarium; the Honors Night Dinner at which four Awards and five Fellow Memberships will be given; and a Symposium on Space Flight with a panel of 12 speakers.

The Annual Business Meeting will occur on Wednesday, December 2, at 10:00 a.m., to be followed at 12:15 p.m. by the Section Luncheon (open to members of all Sections—an informal "get-together").

Those attending technical sessions must register at the ARS Registration Desk on the 24th floor of the McAlpin Hotel, Broadway and 34th St., New York, N. Y. There is no fee for members of the ARS or the ASME. For all others, a fee of $5 will be charged for admission to one or all of the technical sessions. Payment of registration fee is not required for attendance at the luncheon, dinner, or tour.

The program follows:

WEDNESDAY, DECEMBER 2

9:00 a.m.

Registration

9:30 a.m.

Session I

Rocket and Turbine Testing

Chairman: William Smith, Bell Aircraft Co., Buffalo, N. Y.

Vice-Chairman: R. D. Rinehart, Bell Aircraft Co., Buffalo, N. Y.

Experimental Testing Techniques of Extremely High Thrust Rocket Propulsion Systems, by R. F. Gompertz, Edwards AFB, Edwards, Calif. (Preprint no. 104-53)

Multidirection Vibration Tester, by A. Bohr, Reaction Motors, Inc., Rockaway, N. J. (Preprint no. 106-53)

Structural Design Considerations for a High Altitude Sounding Rocket of the Viking Type, by R. Lea, The Glenn L. Martin Co., Baltimore, Md. (Preprint no. 115A-53)

2:30 p.m.

Session II-A

Rocket and Turbine Testing

Chairman: S. K. Hoffman, North American Aviation, Inc., Downey, Calif.

Vice-Chairman: G. P. Sutton, North American Aviation, Inc., Downey, Calif.

Introduction to the Analysis of Supersonic Ramjet Power Plants, by G. Sears and B. Marsh, Marquardt Aircraft Co., Van Nuys, Calif. (Preprint no. 103-53)

The 350,000-Pound Rocket Test Stand at Lake Denmark, N. J., by B. N. Abramson, D. S. Brandwein, and H. C. Menes, U. S. Naval Air Rocket Test Station, Lake Denmark, N. J. (Preprint no. 108-53)

The Problem of Cooling a Rocket Flame Detector, by T. F. Reinhardt, U. S. Naval Air Rocket Test Station, Lake Denmark, N. J. (Preprint no. 107-53)

Session II-B

Rocket Motor Design

Chairman: K. Stehling, Forrestal Research Center, Princeton University, Princeton, N. J.

Vice-Chairman: W. A. Webb, Aircraft Armaments, Inc., Baltimore, Md.

Some Aspects of Design and Fabrication of Liquid Propellant Engines, by H. Davies, Reaction Motors, Inc., Rockaway, N. J. (Preprint no. 111-53)

Rocket Preliminary Design and Proposal Preparation, by S. Lehrer, Reaction Motors, Inc., Rockaway, N. J. (Preprint no. 112-53)

Stability of Ethylene Oxide, by E. M. Wilson, Aerojet-General Corp., Azusa, Calif. (Preprint no. 105A-53)

7:30 p.m.

Hayden Planitarium Tour

A special tour of the Hayden Planetarium, followed by a lecture in the Solar System Room at 8:30 p.m.

THURSDAY, DECEMBER 3

9:30 a.m.

Session III-A

Thermodynamics, Fluid Flow, and Heat Transfer

Chairman: M. W. Rosen, Naval Research Laboratory, Washington, D. C.

Vice-Chairman: R. Snodgrass, Naval Research Laboratory, Washington, D. C.

The Isothermal Compressibilities of Some Rocket Propellant Liquids and the Ratios of Specific Heats, by G. Kretschmar, U. S. Naval Ordnance Test Station, Inyokern, China Lake, Calif. (Preprint no. 100-53)

One-Dimensional Steady Adiabatic Flow in a Constant Area Channel with Mass Addition at Constant Enthalpy and Negligible Kinetic Energy, by E. Price, U. S. Naval Ordnance Test Station, Inyokern, China Lake, Calif. (Preprint no. 101-53)

Investigation of Boiling Heat Transfer and Burnout to JP-4, by C. Beighley and L. Dean, Bell Aircraft Corp., Buffalo, N. Y. (Preprint no. 114-53)

Session III-B

Combustion Studies

Chairman: W. M. Mebane, Thiokol Corp., Huntsville, Ala.

Vice-Chairman: R. Swann, Ordnance Missile Laboratory, Redstone Arsenal, Huntsville, Ala.

Study of the Combustion of Fuel Droplets Descending Through an Oxidizing Atmosphere, by D. Charvonia, Purdue University, Lafayette, Ind., and H. Wood, Virginia Polytechnic Institute, Blacksburg, Va. (Preprint no. 121-53)

Ignition and Combustion in a Laminar Mixing Zone, by F. Marble and T. Adamson, California Institute of Technology, Pasadena, Calif. (Preprint no. 123-53)

Erosive Burning of Some Composite Solid Propellants, by L. Green, Areojet-General Corp., Azusa, Calif. (Preprint no. 122A-53)

2:30 p.m.

Session IV-A

Liquid Properties, Handling, and Analytical Procedures

Chairman: J. Preston Layton, Forrestal Research Center, Princeton University, Princeton, N. J.

Vice-Chairman: Brooks T. Morris, Marquardt Aircraft Co., Van Nuys, Calif.

Oxidant Pumps, by W. Mizen, Bendix Aviation Corp., South Bend, Ind. (Preprint no. 99-53)

An Analytical Procedure for Mixed Acid, by J. Clark, U. S. Naval Air Rocket Test Station, Lake Denmark, N. J. (Preprint no. 119-53)

Experience with the Application of Hydrogen Peroxide for Production of Power, by H. Walter, Worthington Pump & Machinery Corp., New York, N. Y. (Preprint no. 120-53)

Session IV-B

Combustion Studies

Chairman: A. Nerad, General Electric Co., Schenectady, N. Y.

Session IV-B (*Cont.*)

Vice-Chairman: John E. Scott, Jr., Princeton University, Princeton, N. J.

Low-Frequency Combustion Stability of Liquid Rocket Motor with Different Nozzles, by Sin-I Cheng, Princeton University, Princeton, N. J. (Preprint no. 110A-53)

Low-Speed Combustion Aerodynamics, by R. A. Gross and R. Esch, Harvard University, Cambridge, Mass. (Preprint no. 113A-53)

Instantaneous Rocket Flame Temperature Measurement, by John Hett, New York University, New York, N. Y. (Preprint no. 102A-53)

7:00 p.m

Honors Night Dinner

Speaker: Major General Donald L. Putt, U. S. Air Force, Commander, Air Research and Development Command

Subject: Long-Range Applications of Rocket Power

Awards:

R. H. Goddard Memorial Lecture Award to David A. Young, Aerojet-General Corp.

C. N. Hickman Award to C. E. Bartley, Grand Central Aircraft Co.

G. Edward Pendray Award to H. S. Tsien, Robert H. Goddard Professor, California Institute of Technology

Student Award to Alfred D. Goldenberg, Palm Springs, Calif.

Fellow Members:

Luigi Crocco, Princeton University

C. C. Furnas, Cornell Aeronautical Laboratory

Robert L. Gibson, Applied Physics Laboratory

J. W. Mullen, Experiments, Inc.

William Smith, Bell Aircraft Co.

FRIDAY, DECEMBER 4

9:30 a.m.

Symposium on Space Flight

Main Speaker: John R. Pierce, Bell Laboratories, Inc.

Panel: Jacobus M. Botje, Purdue University; Col. J. H. Dewitt, Jr., WSMTV; E. E. Francisco, White Sands Proving Ground; George E. Moore, General Electric Co.; Richard W. Porter, General Electric Co.; Darrell C. Romick, Goodyear Aircraft Corp.; Milton W. Rosen, Naval Research Laboratory; Kurt R. Stehling, Forrestal Research Center; A. R. Tocco, Hughes Aircraft Co.; Comdr. R. C. Truax, USN; Paul F. Winternitz, New York University; and John R. Youngquist, The Glenn L. Martin Co.

Fourth IAF Congress at Zurich Attracts More Than 200 Scientists

ARS Participates

AMERICAN Rocket Society delegates were among the more than 200 scientists who attended the Fourth International Astronautical Federation Congress held on August 2–8, 1953, at Zurich, Switzerland. The Schweizerische Astronautische Arbeitsgemeinschaft, the host society, had arranged for the facilities of Zurich's famed Eidgenossische Technischen Hochschule, made available for the organizational and business meetings as well as for the technical sessions.

Societies in Great Britain, Sweden, Denmark, Germany, Holland, Italy, Argentina, and the United States were represented by more than 200 delegates and member guests. Three new societies from Yugoslavia, South Africa, and Philadelphia, Pa., were admitted to active membership as a result of IAF stimulus. Egypt, Japan, Israel, and a Spanish group from Barcelona sent representatives who announced their intention to further organize their societies and apply for membership in the IAF during the coming year. The total membership of all societies represented was over 6000.

The IAF now consists of astronautical and rocket societies from four continents. Its primary concern is to promote, but not direct, the achievement of space flight and to act as an international clearinghouse for professional thinking directed toward that aim. The IAF has recognized as one of its primary tasks the stimulation of the formation and growth of rocket and astronautical societies throughout the world.

Plenary Sessions

F. C. Durant, III, Named President of IAF

At the plenary sessions, Mr. Frederick C. Durant III, ARS President, was unanimously elected President of the IAF, succeeding Dr. Eugen Sänger, who presided. Mr. Andrew G. Haley, ARS General Counsel, and Prof. Friedrich Hecht of Vienna were similarly chosen as Vice-Presidents.

Other business transacted during these sessions included the decision to complete both an astronautical dictionary and an astronautical library classification system. The Italian Society will continue to compile the dictionary which will eventually appear in French, German, Italian, and English, while a group under Rolf Engel of Cairo, Egypt, will proceed with classification. Another group will be engaged with problems of abstracting papers appearing in books and journals relating to astronautics.

Perhaps of primary importance was the decision to advance, through the activities of a special five-man committee, toward the publication of a proposed quarterly Journal of the IAF. This Journal would contain the results of annual congressional sessions, papers presented during technical periods, news and notes, and such other material deemed appropriate by an editorial board.

Since the need for world scientific recognition of the IAF has been recognized as important, the possibility of direct or indirect affiliation with UNESCO has been and will continue to be explored. The delegates agreed to award periodically a Loeser medal for outstanding achievements in astronautics. The IAF will also sponsor an award for the best paper on the subject, "The Economics of Space Flight."

Scientific Sessions

Six ARS Papers Presented

Among the 33 technical papers presented at the scientific sessions were the following five contributions from ARS members.

Milton W. Rosen and Richard B. Snodgrass, Naval Research Laboratory, Washington, D. C., in presenting their paper "Margin for Error," showed that high altitude rockets never do what calculations suppose. Knowing this, the authors do not believe that present-day plans for satellite vehicles and interplanetary rockets leave sufficient margin for error, and that failure would probably result. They do not wish to dampen optimism but try merely to indicate some shortcomings, based on Viking experience, of the more publicized large rocket and space station proposals.

Kurt R. Stehling, Forrestal Research Center, Princeton University, Princeton, N. J., offered a paper on earth scanning techniques for an orbital rocket, in which both optical-video and X-band radar systems were compared.

Marcel J. E. Golay, U. S. Army Squier Signal Laboratory, Fort Monmouth, N. J., contributed a paper on the application of radio interferometry to interplanetary guidance of rocket ships.

Cmdr. Robert C. Truax, U. S. Navy, forwarded a plan of action to advance the day of space flight. Believing interplanetary travel can be achieved in our lifetime, Cmdr. Truax asserted that the public must be educated to the importance and practicability of such a project.

Wernher von Braun, Guided Missile Development Group, Redstone Arsenal, Huntsville, Ala., in his paper replied to several of the criticisms of his ambitious concepts and studies on satellite vehicles and manned space flight. In particular, Dr. von Braun pointed out the importance of increased reliability of components which would follow a much needed standardization in United States rocket programs. The reduction of rocket engine and other component designs to a few, and the mass production of these would inevitably result in higher reliability. He cited several advances in modern technology that have resulted from such procedures.

Other papers dealt with speculations on photon rockets, nuclear propulsion, dynamics of orbital motions, design of rocket ships, multistage rockets, re-fueling in flight, and other topics.

All technical and general papers presented at the IAF Congress will be published shortly in a bound volume. Details concerning price and availability will appear in a later issue of the JOURNAL.

At the invitation of the Austrian Society, the Fifth International Astronautical Congress will be held on August 2–7, 1954, at Innsbruck, Austria.

Contributed by F. I. Ordway, III

JAMES J. HARFORD, RECENTLY APPOINTED ARS EXECUTIVE SECRETARY

ARS Announces Appointment of Executive Secretary

THE ARS Board of Directors, at its meeting of September 14, 1953, appointed Mr. James J. Harford as Executive Secretary of the national organization.

Creation of this post has been considered for some time, since it was felt that a coordinated program of expansion was necessary for ARS. Mr. Harford, an engineering graduate of Yale University, has had considerable editorial and promotion experience both in Europe and the United States, and is a former Associate Editor of Modern Industry magazine. He will be contacting the various ARS Sections in connection with his activities, and the Board earnestly requests that all ARS members cooperate with him in his efforts.

Mrs. A. C. Slade, whose tireless and enthusiastic work on behalf of the Society has been an enormous factor in its rapid growth, will continue as National Secretary and will work closely with Mr. Harford in implementing the many expansion plans.

Prof. Tombaugh Host to New Mexico-West Texas Section

THE July 17 meeting of the New Mexico-West Texas Section was held at the home of Prof. Clyde W. Tombaugh and his wife in Las Cruces, N. Mex. ARS members and guests were treated to an informal supper followed by an interesting and instructive evening. In spite of a typical New Mexico thunder shower, Prof. Tombaugh, who is with the Flight Determination Laboratory, WSPG, unlimbered his telescopes for the benefit of the group.

Later in the evening, Prof. Tombaugh brought out his large collection of astronomical slides and presented, as he put it, "a two-bit tour of the universe." Many of the slides were unusual in that they have not been generally published, being the work of Prof. Tombaugh while at Lowell Observatory. Since he is the only living man to have discovered a major planet of the Solar System, he presented a thorough description of the methods he used in finding the planet Pluto. In conjunction with this, Prof. Tombaugh showed slides picturing the telescope with which the work was done—the "blink comparator" on which the planet was finally picked out from among the innumerable stars of the background—and direct copies of the original discovery plates themselves.

During the course of the search for Planet X, Prof. Tombaugh covered a major portion of the sky with the telescope camera and succeeded in discovering a number of new nebulae and star clusters. From the appearance of the star plates (which looked like a pound of grain sugar scattered on a piece of black velvet) it seemed miraculous that even one of those tiny grains could be identified as a planet and not a star.

Southern California Section Holds First Fall Meeting

FOLLOWING summer suspension of Section activities, the Southern California Section held its first fall meeting on September 24, 1953, at the Institute of the Aeronautical Sciences, Los Angeles, Calif.

The principal speaker of the evening was Dr. Fritz Zwicky, professor of astrophysics at the California Institute of Technology, and chief research consultant at the Aerojet-General Corporation, Azusa, Calif. His topic was "Scintillations of Stars and Indirect Meteorology," dealing with the measurements of winds in the upper atmosphere. (See pp. 370–372 in this issue of the JOURNAL.)

Dr. Zwicky, a Fellow of the ARS, is an outstanding authority in many fields of science. He pioneered in the theoretical study of physics in the solid state, and is renowned for his astrophysical researches. He has been associated with rocket research in the United States for many years, and is the inventor of many jet propulsion devices.

ARS Vice-President Haley Addresses Northeastern New York Section Meeting

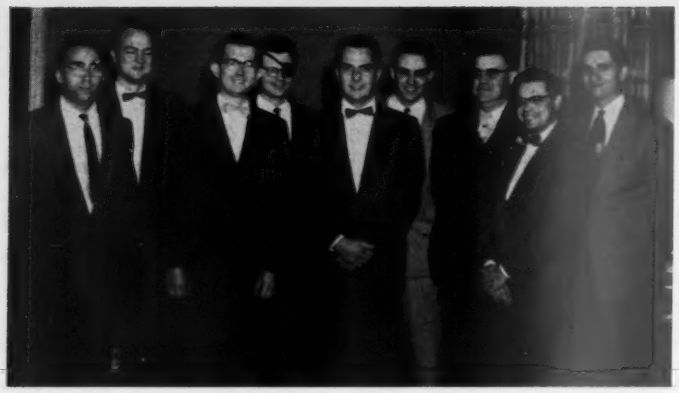
SPEAKING at the September meeting of the Northeastern New York Section, Mr. Andrew G. Haley, ARS Vice-President and General Counsel, told a gathering of 70 members and guests about early American rocket development as exemplified by the Aerojet Engineering Corporation. During this early period in rocket history, he said, courage, vision, and pertinacity played major roles in both the formation of Aerojet and in the development of rockets. The same requisites, he added, are of prime importance to today's rocket engineer.

Mr. Haley also discussed the recent Fourth IAF Congress held at Zurich, Switzerland, of which he is Vice-President. Mr. Haley believes the main accomplishment of the IAF Congress was in the greater degree of stability achieved.

Mr. Haley continued with excerpts from his forthcoming publication on the legal aspects of space flight. He mentioned that navigation in space will probably be governed by rules comparable to those governing navigation in international waters today. In addition, Mr. Haley believes that possession of land areas in space would not depend solely upon landing or settlement but also upon actual direction and support.

A cut-away model of a combustor was the central attraction of a display arranged by the Section's Membership Committee. The display drew much attention and created additional interest in the Section and its activities.

PROF. TOMBAUGH (FIFTH FROM LEFT) DISPLAYS HIS 12-IN. REFLECTING TELESCOPE TO ARS MEMBERS AND GUESTS AT THE JULY 17 MEETING OF THE NEW MEXICO-WEST TEXAS SECTION



OFFICERS OF THE CHICAGO SECTION WITH GUEST SPEAKER, KURT R. STEHLING (*center*), AT THE SEPTEMBER 24, 1953, MEETING

Left to right: S. Altshuler, treasurer; V. J. Cushing, vice-president; D. Ljubenko, secretary; K. Jacobs, president; and Kurt R. Stehling; L. B. Griffis, J. Bryson, S. Coburn, and G. Nothmann, members of the Board of Directors

ARS Newly Formed Chicago Section Has Successful Meeting

THE initial general meeting of the Chicago Section, the latest group to join the American Rocket Society, was held on September 24, 1953, before a large and enthusiastic audience. Forty members plus an additional 90 guests gathered to hear the speaker for the evening, Kurt R. Stehling of the Department of Aeronautical Engineering, Princeton University, Princeton, N. J. An informal dinner attended by the Section officers and Board of Directors, preceded the program.

Mr. Stehling had planned originally to present a sound film reviewing German and European rocket activities of the past 30 years. But this film, which had been shown at the Fourth IAF Congress in Zurich in August, was still in transit at the time of the meeting. Mr. Stehling therefore substituted a talk on high altitudes and interplanetary rockets which was received with great interest.

The officers and Board of Directors of the Chicago Section are as follows: *President,* K. Jacobs; *Vice-President,* V. J. Cushing; *Secretary,* D. Ljubenko; *Treasurer,* S. Altshuler. *Board of Directors:* J. Bryson, S. Coburn, L. B. Griffis, J. Fitzgerald, G. Nothmann, and S. Raynor.

Alabama Section Holds Three Successful Meetings

THE Alabama Section reports three recent and well-attended meetings as evidence of an ever-growing interest in Society activities in that territory.

On August 4, Conrad Swanson addressed the Section on "Ideal Solid Propellants." Certain aspects of the subject were classified as confidential. Mr. Swanson is Chief of the Interior Ballistics Section of the Rocket Development Group, Ordnance Missile Laboratories, Redstone Arsenal.

On September 8, Mr. Lawrence Edgar, Northrop Aircraft representative at the Ordnance Guided Missile School, Redstone Arsenal, spoke on the use of sail planes in scientific research, and in particular as used with the mountain wave project. This work is being conducted by the Air Force Cambridge Research Center, the field work being in the area of the Sierra Mountain Range. The purpose of the project is to study the air flow over mountains and its implications to aviation. During the field experiment two world records for gliders were set when one of the Pratt-Read sail planes climbed to 34,426 ft above the point of release and reached an altitude of 44,255 ft above sea level.

On September 17, Mr. Robert Lusser gave a lecture on "How to Attain Reliability in Guided Missiles." Some aspects of this subject were classified as confidential. The speaker was formerly head of the V-1 program in Germany during World War II. He was at NAMTC, Point Mugu, until March 1953, when he joined the Ordnance Corps as reliability coordinator for guided missiles. His reports on reliability constitute somewhat of a "Bible" on the subject.

Are You Missing Something?

Do you know that an annual subscription to the Journal of the British Interplanetary Society can be obtained for $1.00? By a special exchange arrangement with the BIS, these Journals can be ordered by ARS members from the New York office.

This publication contains original technical and scientific articles on all problems associated with space flight as well as related news items and literature abstracts.

Some 1953 subscriptions are still available, as well as a few back volumes. A list of immediately available issues will be sent upon sequest.

Address: Secretary,
American Rocket Society
29 West 39 Street
New York City, 18.

ARS to Participate in IAS Annual Meeting

The Institute of the Aeronautical Sciences will hold its 22nd Annual Meeting from January 25 through January 29, 1954, at the Hotel Astor, New York, N. Y. The five-day schedule is planned in cooperation with other engineering groups. The Honors Night Dinner is to be held on the evening of January 25.

The ARS-IAS joint rocket propulsion session will take place on Friday, January 29, and three technical papers are to be presented by ARS members. A detailed account of the joint session will appear in the next issue of the JOURNAL.

New York Section Sees Rocket Firings

MORE than 200 members of the New York Section and their guests journeyed to Rockaway, N. J., on September 26 to participate in a program of activities sponsored jointly by Reaction Motors, Inc., and the U. S. Naval Air Rocket Test Station at Lake Denmark.

The day was featured by two test-stand firings: one of a 1000-lb thrust solid propellant rocket and the other of a 1500-lb thrust liquid propellant unit. These took place in the NARTS test area which was later inspected by the gathering. The program opened at NARTS where the group was welcomed by Captain M. L. Leon, USN, commanding officer of the station. P. F. Winternitz, president of the New York Section, introduced speakers who included Thomas F. Reinhardt, Chief Engineer at NARTS, and Harry Horne, Manager, Engineering and Research, of Reaction Motors.

Reinhardt described the activities of NARTS while Mr. Horne told the story of the beginnings of RMI and the building of successful engines, including those which went into the D-558 and the Viking. Then Mr. Horne showed a film on high-speed flight research.

Another interesting session of the New York Section took place on October 23 at the Engineering Societies Building in New York City, when John Evans of the Sperry Gyroscope Company, delivered a talk on Doppler radar. A model unit was on display and evoked considerable interest on the part of the members. Mr. Evans was introduced by James Wheeler, Department Head, Engine Instruments, also of Sperry.

When ordinary temperature limits are exceeded...

CERAMIC COATING

OF PARTS MADE OF

IRON — STEEL
STAINLESS — INCONEL
HASTELLOY
SHOULD BE CONSIDERED.

Write for complete information and prices.

BARROWS PORCELAIN ENAMEL CO.
LANGDON RD. & PENN. R.R., CINCINNATI 13, O.

BARROWS B P PORCELAIN

Book Reviews

H. S. SEIFERT, California Institute of Technology, *Associate Editor*

MAN IN SPACE, by Heinz Haber, The Bobbs-Merrill Company, Inc. N. Y., 1953, 291 pp. $3.75.

Reviewed by K. BUETTNER
University of Washington, Seattle, Wash.

The advance in rocketry has brought us closer to the conquering of the last border, that of space. The crushing events connected with the development of the V-2 have made the discussion of possible space flight widespread. The human aspect of future space flight has been considered an important part of any such project. Important scientific meetings have been held that were more or less solely concerned with the aspect of man in flights beyond the atmosphere. The San Antonio, Texas, meeting in 1950 has been fully described in "Physics and Medicine of the Upper Atmosphere," edited by the University of New Mexico. Such articles, however, are not meant for easy reading. Many popular books on space flight include parts about the human aspect; however, they are not written by the investigating scientist and are not always dependable. Only few of them, for instance, recognize the rather interesting fact that weightlessness or absence of gravitational pull is not restricted to very large distances from the earth.

It is this topic which made Heinz Haber a well-known figure in the new science of space medicine. He has written the book on this field which is not only correct and of a high scientific standard, but which makes also easy reading. The basis of space flight from its engineering side is mainly based on the calculations of H. Oberth, W. von Braun, etc. Preference is given to the creation of an artificial satellite before any fancy landings on the Moon or Mars. The ascent through the friction layers of the lower atmosphere and the extreme vacuum of the outer space confront us with many physiological, psychological, and technical problems, such as high acceleration, high compartment temperatures, and some housekeeping difficulties. The most serious problems, however, will be met for a crew cruising over the earth in a stable celestial orbit. We are far from a solution of the psychological and physiological questions with respect to weightlessness. We have to be careful in maintaining a white painting of the outer wall to prevent the ship from being overheated by solar rays. We are still not sure that the cosmic ray primaries may not do harm to the nuclei of our body cells and may not sometime suddenly damage important small areas of the human bodies, such as certain nerve centers in the brain and the fovea of the eye. The large number of small meteorites will certainly scratch off the white paint and sandblast any openly exposed optical surfaces, whereas the more infrequent large meteors might bring the journey to a sudden end.

The value of this book reaches beyond its main topic. In view of its clarity of definition on the one side and its easy readability on the other side, it makes an excellent educational textbook.

SMALL PARTICLE STATISTICS, by G. Herdan, Elsevier Publishing Company, New York, 1953, 528 pp. $12.

Reviewed by S. DOBRIN
California Institute of Technology
Jet Propulsion Laboratory

Considering that there are so many branches of science and technology which are concerned with materials in the finely divided state, it is unfortunate that the literature on the characteristics of small particles has been so sparse. Knowledge of the particle size, particle shape, and size distribution of such materials as pigments, clays, cements, solid propellant oxidizers, catalysts, adsorbents, reagents, soil deposits, food products, sprays, smokes, emulsions, and dispersions is of great aid to the people working with them, but in many cases, particularly where particles below 40 microns are concerned, no adequate means of quantitative measurement exist.

Herdan, by means of statistical mathematics, undertakes to bridge the gap between the type of information desired about systems of small particles and the available experimental methods of measurement. He states that, "for the correct appreciation of the influence which particle size and other properties of particle(s) . . . exert upon the physical macroscopic characteristics of materials, some knowledge of correlation theory appears to be indispensable. Vagueness, lack of confidence in established relationships, and reluctance to draw from extensive experimental results any but the most obvious conclusions . . . these are the consequences of not using the appropriate methods of analysis."

A large portion of the book deals with methods of data interpretation, graphical and functional representation, deviations, errors, and correlation techniques. Except that functions are adapted to characteristics of small particles, the book does not differ from other texts on statistical methods. But it also treats of distribution functions which may apply to systems of small particles, and defines in mathematical terms the various average particle sizes which are used to characterize the distributions.

The methods of particle-size analysis are discussed, and throughout the book are references to particle-size measurements on various substances. There are chapters on mixing, on grinding, and on the molecular weights of high polymers, but these chapters are rather inadequate.

The book has probably been published prematurely. Not that the author has not organized his subject well—for he has—but the spread between desired information and available measurement techniques is still too great to be filled by statistical analysis of data.

Advances in the science of small particles still await the development of direct or indirect methods for measuring the sizes, shapes, and quantities of particles in the subsieve range.

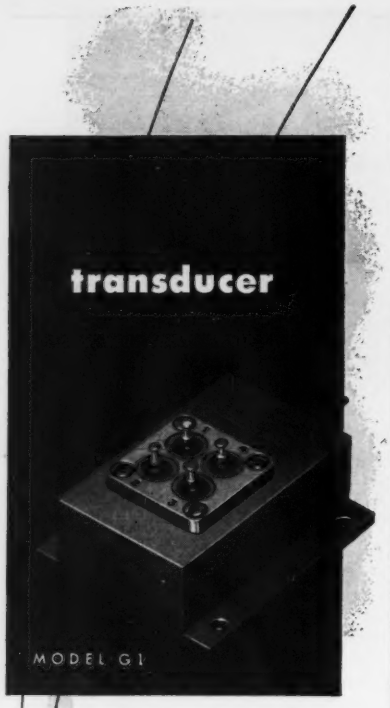
CEREBRAL MECHANISMS IN BEHAVIOR, THE HIXON SYMPOSIUM, L. A. Jeffress, editor, John Wiley & Sons, Inc., New York, 1951, 311 pp. $5.20.

Reviewed by R. M. STEWART
California Institute of Technology
Jet Propulsion Laboratory

This book contains a collection of papers (and discussions of the papers) presented at a symposium held at the California Institute of Technology in September 1948. Those who work either directly or indirectly with automatic computing and control equipment will find this book very interesting, since a large part of the discussion is devoted to speculations concerning the similarity of automatic computing mechanisms to human cerebral mechanisms.

The first paper, which epitomizes the conclusions of the group, is by John von Neumann and is entitled The General and Logical Theory of Automata. Having been written by one of the outstanding designers of large-scale digital computers, it should especially appeal to other workers in this field. He points out three factors which prevent us at the present time from designing even conceptually a mechanism which can imitate the performance of the human brain: (1) The necessary size, using any known techniques for performing elementary switching functions, would be stupendous. (2) It is impossible, using present computer techniques, to obtain the required system reliability in spite of failure of components. (3) We are very far from possessing a theory of automata which deserves that name.

Von Neumann describes the McCulloch-Pitts theory of neutral networks which is based entirely on a few similar types of elementary switching organs, but he criticizes it primarily in the way in which learning and memory are handled. According to the McCulloch-Pitts theory, memory is accomplished by means of closed loops of reverberating memory circuits. Although there is some direct evidence for "reverberative memory," most neurologists feel that memory is primarily due to some permanent or semipermanent physical or chemical changes in the nervous system. Also, von Neumann expresses doubt that such a system would be sufficiently economical in the number of switching organs required to be able to be fitted into the head (or possibly, the universal!). Other members of the symposium take an even more skeptical view of such theories, maintaining that



The Model G1 transducer is a research tool designed for the measurement of forces from ± 1.5 ounces to ± 80 ounces and for the measurements of displacements to ± 0.0015 inch.

Since the moving parts of the transducer weigh but 4.3 grams, the dynamic characteristics of most systems to which it is coupled are relatively unchanged.

Force or displacement applied to the transducer probe is translated into an exact electrical equivalent by means of a complete balanced bridge of strain sensitive resistance wire.

Please request Bulletin No. 1.1.

S-L Statham LABORATORIES • Los Angeles 64, Calif.

cerebral mechanisms can be understood only in terms of a "field theory" and that the "all-or-none" concept of nerve impulses, which is basic to the McCulloch-Pitts theory and to digital-computing techniques, is only incidental to the primary mechanisms involved. Most of the evidence supporting a "field theory" seems to relate to visual perception.

All participants of the symposium agree that the problem under discussion is far from being solved.

EXTERIOR BALLISTICS, by E. J. McShane, J. L. Kelley, and T. V. Reno, The University of Denver Press, Denver, Colo., 1953, 834 pp. $10.80.

Reviewed by E. P. BAXTER
California Institute of Technology
Jet Propulsion Laboratory

This book, a result of the authors' work at Aberdeen during World War II, collects into a single volume the information needed to calculate the trajectories of shells, bombs, and rockets. It covers, in varying degrees of detail, the theoretical bases of ballistic calculations, the experimental determination of the necessary data, and the techniques of numerical computation, ending with an interesting historical appendix.

After an introductory chapter of 152 pages that covers vector analysis, the dynamics of rigid bodies, dimensional analysis (including a rigorous proof of the Buckingham π theorem), and statistics, the authors derive the equations of motion of a spinning projectile over a rotating earth. The next chapter discusses methods of measuring aerodynamic forces and moments. Then follows a development, in historical order, of the approximate equations that are still useful in artillery calculations and of their extensions to antiaircraft and aircraft artillery bombs. The applicability and limitations of each method are clearly stated. A section on ballistic and firing tables discusses the organization of the tables and the best choice of parameters for them.

Methods of numerical integration of the equations are covered in detail with careful attention to the estimation of errors, and the methods are applied in a series of examples to the calculation of normal trajectories. Three chapters are devoted to the theory and computation of differential corrections to the normal trajectory, again with examples worked out. The contents of the remaining chapters of the book are indicated by their titles: Bombing, Angular Motion of a Projectile, Launching Effects, Reduction of Spark Range Data, and Rockets.

This book should prove a very useful guide to anyone wishing to compute trajectories for any sort of missile. Although the amount of detail devoted to different subjects varies considerably, there are many references to more complete treatments of the subjects which are not thoroughly discussed.

A note should be added on the format. Although the book is lithoprinted from a typewritten manuscript, the equations are very well set up, so that the book is easy to read.

THERMODYNAMICS OF FLUID FLOW, by N. A. Hall, Prentice-Hall, Inc., New York, 1951, 278 pp. $7.35.

Reviewed by D. ALTMAN
California Institute of Technology
Jet Propulsion Laboratory

The increasing interest during recent years in various phases of fluid flow has given rise to many treatments in the literature of various aspects of this field. The work of summarizing and condensing certain phases of this work in book form is therefore a justifiable project. The author is professor of mechanical engineering at the University of Minnesota and has presumably been teaching portions of this subject matter in his courses.

There are 278 pages in the text which is divided into twelve chapters bearing the following titles: Introduction, Fluid Friction, Energy and Continuity, Momentum Principles, Incompressible Fluids, Perfect Gases, Constant-Area Adiabatic Flow, Varying-Area Adiabatic Flow, Shock Regions, Flow Measurement and Control, Diabatic Flow, and Propulsion Systems. There are several appendixes of which one contains 62 pages of tables listing dimensionless-flow parameters for various situations. Several small errors in the book include an oversight in the last two columns of Table D.1 in which the squares of the functions listed have been tabulated.

The treatment is based mainly upon two major assumptions, that the flow is steady and that it is one-dimensional. These assumptions greatly simplify the analysis and, in more complicated flow patterns, provide a preliminary estimate of the flow parameters. The rather simplified approach and the extensive use of examples and problems make the text more suitable for the student in the classroom than for the research engineer in the field. The author has included many of the classical problems in the field, although he has added very few of his own interpretations. The subject matter actually treated is quite limited in scope; if the examples and problems were eliminated and the number of equations reduced to the significant ones, the book would be decreased very materially in size. Omitted are such subjects as flow with mass addition and two-phase flow. Only two pages are devoted to the subject of rockets; without appearing subjective, this reviewer believes that current interest in this field could warrant a somewhat larger treatment.

VISION THROUGH THE ATMOSPHERE, by W. E. Knowles Middleton, University of Toronto Press, 1952, 250 pp. $8.50.

REVIEWED BY F. E. LITTMAN
Stanford Research Institute, Pasadena, Calif.

Dr. Middleton's discussion of the "visibility problem" is a competent and readable attempt to gather up the many factors pertinent to this interesting field, determine the contribution of each, and thus present an accurate picture of the over-all situation.

Starting with a brief discussion of elementary photometric theory, the book continues in a logical fashion with an examination of the optical properties of

the atmosphere and the relevant properties of the human eye. Thus the groundwork is laid for a discussion of the visual range of objects and light sources. By pointing out the shortcomings of subjective observations of the "visibility," a strong case is made for an alternative, the instrumental measurement of the extinction coefficient. This measurement could be translated into something resembling the visual range as used by the meteorologist, but would not be encumbered with the uncertainties inherent in visual observations under widely varying conditions.

The book concerns itself mostly with the physical and psychological basis of the problem. It is written not as a handbook for meteorological observers but rather as a source of information from all fields contributing to this complex problem. The thoroughness of the presentation and the author's lucid style make its reading both profitable and enjoyable.

Books

Tables for Rocket and Comet Orbits, by S. Herrick, U. S. Government Printing Office, Superintendent of Documents, Washington 25, D. C., March 9, 1953, 100 pp. $1.75.

Hydrazine, by C. C. Clark, Mathieson Chemical Corp., Baltimore, Md., 1953, 133 pp.

Advances in Applied Mechanics, Volume III, by R. von Mises and T. von Kármán, editors, Academic Press, Inc., N. Y., 1953, 324 pp. $9.

Oxidation of Metals and Alloys, by O. Kubaschewski and B. E. Hopkins, Academic Press, Inc., N. Y., 1953, 239 pp.

Solid State Physics, by C. Kittel, John Wiley & Sons, Inc., N. Y., 1953, 409 pp. $7.

The Physics of the Stratosphere, by R. M. Goody, Cambridge University Press, N. Y., 1953, 192 pp. $4.75.

Papers of the 1953 Heat Transfer and Fluid Mechanics Institute, Stanford University Press, Stanford, Calif., 1953, 240 pp. $5.50.

The Mathematical Theory of Non-Uniform Gases: an Account of the Kinetic Theory of Viscosity, Thermal Conduction and Diffusion in Gases, by S. Chapman and T. G. Cowling, University Press, Cambridge, England, 1952, 431 pp. 60s.

The Radiant Universe, by G. W. Hill, Philosophical Library, Inc., N. Y., 1952, 489 pp. $4.75.

Stars in the Making, by Cecilia Payne-Gaposchkin, Harvard University Press, Cambridge, Mass., 1952, 160 pp., photographs 48 pp. $4.25.

Proceedings of the Second Midwest Conference on Fluid Mechanics, Dynamics of the Opening Parachute, by J. R. Foote and M. G. Scherberg, Ohio State University Press, Columbus, Ohio, 1952, pp. 131–144. $6.

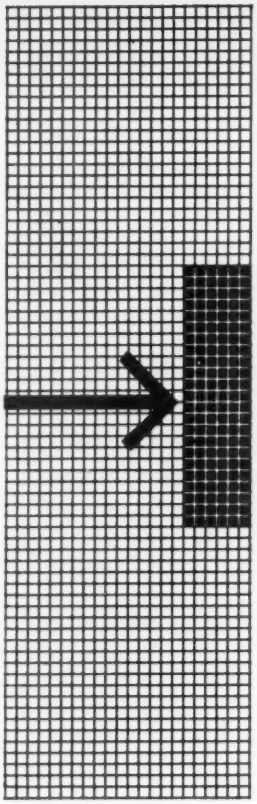
Design of Machine Elements, by M. F. Spotts, Prentice-Hall, Inc., N. Y., 1953, 504 pp. $9.65.

Weight-Strength Analysis of Aircraft Structures, by F. Shanley, McGraw-Hill Book Co., Inc., N. Y., 1952, 394 pp. $8.50.

Evaluating Research and Development, I. R. Weschler and P. Brown, editors, University of California (Institute of Industrial Relations), Los Angeles, 1953, 104 pp. $1.65 (paper).

An Introduction to Scientific Research, by E. B. Wilson, Jr., McGraw-Hill Book Co., Inc., N. Y., 1952, 375 pp. $6.

Electronic Engineering Principles, by J. D. Ryder, Prentice-Hall, Inc., N. Y., 1952, 505 pp. $9.



diversification
at Lockheed in California

means
better careers for engineers

new aerodynamics *and* **thermodynamics positions** *have been created by Lockheed's diversified expansion program*

Already 12 models of luxury airliners, cargo transports, radar search planes, fighters, bombers and trainers are in production.

Already Lockheed's development program is the most diversified in its history.

Now new positions have opened up for:

AERODYNAMICS ENGINEERS
AERODYNAMICISTS "A" & "B"
JR. ENGINEERS (*for aerodynamics work*)
A Degree in Aeronautical Engineering or Mechanical Engineering with an Aero option.

THERMODYNAMICS ENGINEERS
THERMODYNAMICISTS "A" & "B"
JR. ENGINEERS (*for thermodynamics work*)
A Degree in Aeronautical Engineering or Mechanical Engineering with a Thermo, Aero or Power Plant option.

Here is Lockheed's diversification in action. These openings are for work on:

Guided missiles
New fighter designs, such as the FX-104
Special research planes
Research studies
New jet transport designs
Continuing development of current production models

Lockheed invites qualified engineers to apply for these Aerodynamic and Thermodynamic positions. Coupon at right is for your convenience.

Mr. E. W. Des Lauriers
Engineering Recruiting, Dept. R 11
Lockheed Aircraft Corporation, Burbank, California

Dear Sir: Please send me an application form and illustrated brochure describing life and work at Lockheed in California.

my name _____

my field of engineering _____

my street address _____

my city and state _____

Lockheed

LOCKHEED AIRCRAFT CORPORATION, BURBANK, CALIFORNIA

Technical Literature Digest

M. H. SMITH and M. H. FISHER

The James Forrestal Research Center, Princeton University, Princeton, N. J.

Jet Propulsion Engines

The Rotary Regenerative Air Preheater for Gas Turbines, by A. T. Bowden and W. Hryniszak, *Trans. ASME*, vol. 75, July 1953, pp. 767–777.

Ramjet Possibilities, *Flight*, vol. 114, July 3, 1953, pp. 7–12.

Theory and Practice of Gas Turbine Power Plants for Helicopters, by Igor B. Bensen, Part II, *American Helicopter*, vol. 31, June 1953, pp. 10–13, 16.

Tailpipe "teeth" cut jet engine noise, *Aviation Week*, vol. 59, July 6, 1953, pp. 28–29.

The Repair and Overhaul of Gas Turbines, by S. M. Saeed, *Aircr. Engng.*, vol. 25, July 1953, pp. 200–201.

Proposed Airworthiness Policies for Certification of Turbine Powered Transport Airplanes, Civil Aeronautics Administration, Turbine Powered Transport Evaluation Team, Jan. 1953, 57 pp.

The Propulsive Efficiency and Best Division of Power in an Augmented Jet Engine, by John Mitchell Stephenson. *J. Aero. Sci.*, vol. 20, Aug. 1953, pp. 576–577.

Note on the Efficiency of Thermal Machines at a Finite Rate of Expansion, by L. J. F. Broer, *Appl. Sci. Res.*, Section A, vol. 4, 1953, pp. 1–10.

Turbine-Powered Commercial Aircraft, by DeWitt C. Ramsey, *Aero. Engng. Rev.* vol. 12, Aug. 1953, pp. 34–36.

Status of the Turbine-Powered Transport in the United States, by Frederick B. Lee, *Aero. Engng. Rev.*, vol. 12 Aug. 1953, pp. 29–33.

Air-Line Point of View on Status of the Turbine-Powered Transport in the United States by Milton W. Arnold, *Aero. Engng. Rev.*, vol. 12, Aug. 1953, pp. 37–39.

MATS and the Gas-Turbine Transport Field, by Joseph Smith, *Aero. Engng. Rev.*, vol. 12, Aug. 1953, pp. 40–42.

An Experimental Single-Stage Air-Cooled Turbine, by J. Reeman and R. W. A. Buswell, *Aircr. Engng.*, volume 25, Aug. 1953, pp. 227–235.

Cornell Valveless Pulsejet Engine Tests and Further Developments, by C. D. Porter, *Naval Research Lab. Rep.* no. 4186, Aug. 5, 1953, 23 pp.

Turboprops Power Long Range Red Bombers, by Anthony Vandyk, *Amer. Aviation*, vol. 17, Aug. 31, 1953, pp. 27–29.

Turbines à Gaz au Bourget, *Aeroplane*, vol. 85, Aug. 7, 1953, p. 172.

The Viper, *Flight*, vol. 64, Aug. 7, 1953, pp. 170–174.

Air Admixture to Exhaust Jets, by Eugen Sänger, *NACA TM* 1357, July 1953, 35 pp. (translated from *Ingenieur Archiv*, vol. 18, 1950, pp. 310–323).

Some NACA Research on Centrifugal Compressors, by I. A. Johnsen and Ambrose Ginsburg, *Trans. ASME*, vol. 75, July 1953, pp. 805–817.

Periodic Flow Regenerator, by J. E. Coppage and A. L. London, *Trans. ASME* vol 75, No. 5, July 1953, pp. 779–787.

Rocket Propulsion Engines

The Atomic Hydrogen Rocket, by D. J. H. Wort, *J. Brit. Interplan. Soc.* volume 12, July 1953, pp. 167–172.

Heat Transfer and Fluid Flow

Air-Water Analogy and the Study of Hydraulic Models, by Giulio Supino, *NACA TM* no. 1359, July 1953. 23 pp. (translated from *Energia Elettrica*, vol. 28, Nov. 1951.)

Some Tests on Compressor Cascade of Related Aerofoils Having Different Positions of Maximum Camber, by A.D.S. Carter. *Gt. Britain Aeron. Research Counc. Rep. Mem.* no. 2694 (formerly *ARC.* Tech. Rep. 12324, National Gas Turbine Establ. Rep. R.47), 1953, 15 pp. 10 fig.

Propeller Parameters and the Axial Compressor by V. D. Naylor. *Aircr. Engng.* vol. 25, July 1953, pp. 190–193.

Measurements of Heat Transfer and Friction Coefficients for Air Flowing in a Tube of Length Diameter Ratio of 15 at High Surface Temperatures, by Walter F. Weiland and Warren H. Loudermilk, *NACA RM* no. E53E04, July 1953, 19 pp.

Experiments on Mixed Free and Forced Convective Heat Transfer Connected with Turbulent Flow Through a Short Tube, by Ernest R. G. Eckert, Anthony J. Diaguila, and Arthur M. Curren, *NACA TN* no. 2974, July 1953, 59 pp.

Plane Jet of Gas Emitted from a Nozzle with Supersonic Velocity, by Ziro Hasimoto, *J. Phys. Soc. of Japan*, vol. 8, May-June 1953, pp. 394-399.

On the Flow in a High-Pressure Valve, by F. A. Goldsworthy, *Proc. Roy. Soc.*, Ser. A, vol. 218, June 9, 1953, pp. 69–87.

Transport of Momentum, Mass, and Heat in Turbulent Jets, by Lloyd G. Alexander, Thomas Baron and Edward W. Comings, *Illinois Univ. Engng. Exper. Sta. Bull. Ser.* no. 413, May 1953, 88 pp.

Convection Currents in Porous Media, by F. T. Rogers, Jr., V. Variational Form of Theory, *J. Appl. Phys.*, vol. 24, July 1953, pp. 877–880.

Normal Shock Wave Phenomena in a Convergent-Divergent Nozzle, by B. W. Martin, *J. Roy. Aeron. Soc.*, vol. 57, July 1953, pp. 455–460.

Unsteady Supersonic Flow Past Slender Pointed Bodies, by John W. Miles, *NAVORD Report* 2031 (*Naval Ord. Test Sta.* Rep. 701), May 1953, 68 pp.

Supersonic Flow over Conical Bodies Without Axial Symmetry, by Antonia Ferri, Nathan Ness, and Thaddeus T. Kaplita, *J. Aero. Sci.*, vol. 20, Aug. 1953, pp. 563–571.

Turbulent Boundary Layers in Adverse Pressure Gradients, by Francis H. Clauser, *John Hopkins Univ. Dept. Aeron.*, June 1953, 41 pp. 36 figs.

Thermal Convection in Laminary Boundary Layers, by H. J. Merk, and J. A. Prins, *Appl. Sci. Res.*, Sec. A, vol. 4, 1953, pp. 11–24.

Transformation Between Compressible and Incompressible Boundary-Layer Equations, by Nguyen Van Le, *J. Aero. Sci.*, vol. 20, Aug. 1953, pp. 583-584.

Effect of Gas Radiation in the Boundary Layer on Aerodynamic Heat Transfer, by John W. Smith, *J. Aero. Sci.*, vol. 20, Aug. 1953, pp. 579–580.

On the Role of Three-Dimensional Disturbances in the Stability of Supersonic Boundary Layers, by D. W. Dunn, and C. C. Lin, *J. Aero. Sci.*, vol. 20, Aug. 1953, pp. 577–578.

Apparent Recovery Factors on the Leading Edge of a Flat Plate in Supersonic Laminar Flow, by Irving Korobkin, *J. Aero. Sci.*, vol. 20, Aug. 1953, pp. 578–579.

Pressure Distributions on the Blade of an Axial-Flow Propeller Pump, by D. A. Morelli and R. D. Bowerman, *Trans. ASME*, vol. 75, Aug. 1953, pp. 1007–1013.

Comparison of Secondary Flows and Boundary Layer Accumulations in Several Turbine Nozzles, by Milton G. Kofsky, Hubert W. Allen and Howard Z. Herzig, *NACA TN* no. 2989, Aug. 1953, 58 pp.

A Brief Survey of Progress of the Mechanics of Cavitation, by Phillip Eisenberg, *David W. Taylor Mod. Basin Rep.* 842 (Addendum to *DTMB Rep.* 712), June 1953, 39 pp.

Discussions and Author's Closure for a Brief Survey of Progress on the Mechanics of Cavitation, by Phillip Eisenberg, *David W. Taylor Mod Basin, Rep.* 842A, June 1953, 12 pp.

Heat Transfer in Laminar Flow Through Cylindrical Tubes, by J. Schenk and J. M. Dumore. *Appl. Sci. Res.*, Sec. A, vol. 4, 1953, pp. 39–51.

Non-Uniform Heat Transfer to Fluids Flowing in Conduits, by V. J. Berry, Jr., *Appl. Sci. Res.*, Sect. A, vol. 4, 1953, pp. 61–75.

Heat Transfer to Molten Lead-Bismuth Eutectic in Turbulent Pipe Flow, by H. A. Johnson, J. P. Hartnett, and W. J. Clabaugh, *Trans. ASME*, vol. 75, Aug. 1953, pp. 1191–1198.

Comparison of Theoretical and Experimental Heat-Transfer Characteristics of Bodies of Revolution at Supersonic Speeds, by Richard Scherrer, *NACA Rep.* no. 1055, 1951, 15 pp.

The Determination of Skin Temperatures Attained in High Speed Flight, by F. V. Davies, and R. J. Monaghan, *Gt. Brit. Aeron. Res. Counc. Current Paper* no. 123, 1953, 41 pp. 18 figs.

An Analysis of Laminar Free-Convection Flow and Heat Transfer About a Flat Plate Parallel to the Direction of the

EDITOR'S NOTE: This collection of references is not intended to be comprehensive, but is rather a selection of the most significant and stimulating papers which have come to the attention of the contributors. The readers will understand that a considerable body of literature is unavailable because of security restrictions. We invite contributions to this department of references which have not come to our attention, as well as comment on how the department may better serve its function of providing leads to the jet propulsion applications of many diverse fields of knowledge.

Generating Body Force, by Simon Ostrach, *NACA Rep.* no. 1111, 1953, 17 pp.

A Summary of Basic Heat Transfer and Flow Friction Design Data for Plain Platefin Heat Exchanger Surfaces, by W. M. Kays, and S. H. Clark, *Stanford Univ. Dept. Mech. Engng. Tech. Rep.* No. 17, Aug. 15, 1953, 88 pp.

A New Method for Calculating Laminar Heat Transfer on Cylinders of Arbitrary Cross-Section and on Bodies of Revolution at Constant and Variable Wall Temperature, by H. Schuh, *Stockholm Royal Inst. Technology, Tech. Note* no. Aero 33, 1953, 40 pp.

Stability of Plane Poiseuille Flow, by L. N. Thomas, *Phys. Rev.*, vol. 91, Aug. 15, 1953, pp. 780-783.

Self-Excited Oscillations and the Intrinsic Equations of Viscous fluids, by Francois-Joseph Bourriéres, *France, Ministére del'Air Pub. Sci. et. Techniques* no. 279, 1953, 95 pp. (in French).

On the Asymptotic Behaviour of Compressible Fuid Flow at a Great Distance from a cylinder in the Absence of Circulation, by Isao Imai, *J. Phys. Soc. Japan*, vol. 8, July-Aug. 1953, pp. 537–544.

Approximations of Equations Governing the Flow of Gases. Application to Plane Nozzles, by M. Fenain, *Recherche Aéron.* no. 33, May-June 1953, pp. 11–28 (in French).

Visualization in the Hydrodynamic Tunnel, by H. Werle, *Recherche Aéron.* no. 33, May-June 1953, pp. 3–7 (in French).

General Study of the Flow of a Gas Through Any Tube and of the Transition Through the Speed of Sound (Permanent or Varied Regime with Transfer of Heat and Eventual Chemical Reactions), by Max Serruys, *French Ministére de l'Air Pub. Sci. et. Techniques* no. 272, 1952, 62 pp. (in French)

Measurement and Analysis of Turbulent Flow Containing Flow Fluctuations, by William R. Mickelsen and James C. Lawrence, *NACA RM* E53F19, Aug. 1953, 45 pp.

Combustion

The Thermodynamics of Combustion Gases; Temperatures of Methane-Air, Propane-Air, and Ethylene-Air Flames, by Robert W. Smith, Jr., John Manton, and Stuart R. Brinkley, Jr., *Bur. of Mines, Rep. of Investigations*, 4983, June 1953, 21 pp.

Studies on the Spontaneous Ignition of Fuels Injected into a Hot Air Stream. V. Ignition Delay Measurements on Hydrocarbons, by B. P. Mullins, *Fuel*, vol. 32, July 1953, pp. 363–379.

Studies on the Spontaneous Ignition of Fuels Injected into a hot air Stream, IV. Ignition Delay Measurements on Some Gaseous Fuels at Atomospheric and Reduced Static Pressures, by B. P. Mullins, *Fuel*, vol. 32, July 1953, pp. 343–362.

Spontaneous Ignition of Fuels Injected into a Hot Air Stream. III. Effect of Chemical Factors upon the Ignition Delay of Kerosene Air Mixtures, by B. P. Mullins. *Fuel*, vol. 32, July 1953, pp. 327–342.

Analytical (Power Series) Solutions to the Equations of Flame Propagation, by J. O. Hirschfelder, and Edwin S. Campbell, *Wisconsin Univ. Naval Res. Lab.*, CM-784, May 1953, 117 pp.

High Temperature Combustion, by H. Reichert. *Aircr. Engng.* vol. 25, July 1953, pp. 198–199, 212.

The Oxidation of Mixtures of Hydrogen and Carbon Monoxide. Part I. The Second Explosion Limits of Mixtures of Hy-

We helped take the MISS out of GUIDED MISSILES

the problem: The electronic brain which guides a guided missile is as delicate as it is complicated. One of the major problems, therefore, which faced missile engineers was how to protect this sensitive mechanism from the violent shock and vibration set up by the missile at take-off and in flight.

the solution: Since conventional types of shock-mountings proved to be inadequate, Robinson engineers were called in to find a solution. After careful study and intensive laboratory work, Robinson developed a special multi-directional mounting that reduced vibration and shock to such a marked degree that the missile's guidance system was free-floating and thus dependable. Today, these mountings are in quantity production at the Robinson factories . . . *and reliability now rides the rockets!*

Do you have a problem in Vibration Control?

This same engineering know-how and skill can be put to work on *your* vibration-control problem . . . whether it involves precision instruments, electronic or television equipment, aircraft, motor vehicles, home appliances, or machinery of any size or weight.

Robinson Engineered Mounting Systems are built to outlast the equipment to which they are applied. Unlike old-fashioned rubber mountings, Robinson Met-L-Flex* mountings are impervious to age, oil, bacteria, water, dust, dirt or temperature extremes. They are permanently damped; cannot pack down or wear out; maintain full efficiency for their entire lifetime.

Some vibration problems can readily be solved by standard Robinson mounts. Others require special systems to meet unusual conditions.

A letter or telegram will bring a Robinson engineer to analyse your particular problem and suggest a solution, at no obligation to you. *Write or wire us immediately, attention of Dept. AW3.*

ROBINSON AVIATION INC.
TETERBORO, NEW JERSEY
Vibration Control Engineers

drogen, Carbon Monoxide and Oxygen in a Potassium Chloride Coated Silica Vessel, by G. Dixon-Lewis, and J. W. Linnett, *Trans. Faraday Soc.*, vol. 49, July 1953, pp. 756–765.

The Oxidation of Mixtures of Hydrogen and Carbon Monoxide. Part 2. The Second Explosion Limits of Mixtures of Hydrogen, Carbon Monoxide, and Oxygen in a Clean Silica Vessel, by G. Dixon-Lewis, J. W. Linnett, and D. F. Heath, *Trans. Faraday Soc.*, vol. 49, July 1953, pp. 766–771.

Carbon Formation in Acetylene Flames. Calculation of Free Energies of Reaction, by Katherine Hellwig and Robbin C. Anderson, *Texas Univ. Dept. Chemistry*, Combustion Kinetics Project. Tech. Rep. no. 5, May 1953, 20 pp.

Nature of Gas Burner Flash-Back on Turndown, by Joseph Grumer. *Ind. Engng. Chem.*, vol. 45, Aug. 1953, pp. 1775–1776.

Spontaneous Ignition Properties of Fuels and Hydrocarbons by J. Enoch Johnson, John W. Crellin, and Homer W. Carhart, *Ind. Engng. Chem.*, vol. 45, Aug. 1953, pp. 1749–1753.

Tetraethyl Radiolead Studies of Combustion Chamber Deposit Formation, by H. P. Landerl and B. M. Sturgis, *Ing. Engng. Chem.*, vol. 45, Aug. 1953, pp. 1744–1748.

High Temperature Combustion Chamber, by Alexander Weir, Jr., *Ind. Engng. Chem.*, vol. 45, Aug. 1953, pp. 1637–1644.

Combustor Performance with Instantaneous Mixing by W. H. Avery and R. W. Hart. *Ind. Engng. Chem.*, vol. 45, Aug. 1953, pp. 1634–1637.

Flame Stability in Bluff Body Recirculation Zones, by John P. Longwell, Edward E. Frost, and Malcolm A. Weiss, *Ind. Engng. Chem.*, vol. 45, Aug. 1953, pp. 1629–1633.

Interaction of a Plane Flame Front with a Plane Sinusoidal Shear Wave, by George H. Markstein, *J. Aero. Sci.*, vol. 20, Aug. 1953, pp. 581–582.

A Constant-Area, Constant-Temperature Combustor, by Philip Rosen and R. W. Hart, *J. Aero. Sci.*, vol. 20, Aug. 1953, pp. 549–554.

Effects of Self-Absorption on Rotational "Temperatures" of OH in Flames, by Herbert P. Broida, *J. Chem. Phys.*, vol. 21, July 1953, pp. 1165–1169.

Methods of Measuring Combustion Efficiency in Gas Turbine, by J. Fellons, *Rev. Inst. Francaise Pet. Ann. Combustibles Liquides*, vol. 8, June 1953, pp. 277–281 (in French).

Ignition Studies. Part II. The Spontaneous Ignition Properties of Some Diesel Fuels and Pure Hydrocarbons, by J. Johnson, John W. Crellin, and Homer W. Carhart, *Naval Res. Lab. Rep.* no. 4202, July 1953, 14 pp.

Spontaneous Ignition of Lubricating Oils, by Charles E. Frank, Angus U. Blackham, and Donald E. Swarts, *Ind. Engng. Chem.*, vol. 45, Aug. 1953, pp. 1753–1759.

Theory of Laminar Flame Stability, by J. B. Rosen. *Princeton Univ. Chemical Kinetics Project. Tech. Rep.* no. 4, July 1953, 65 pp., 11 figs.

Detection of Atoms and Free Radicals in Flames by Mass Spectrometric Techniques, by S. N. Foner and R. L. Hudson, *J. Chem. Phys.*, vol. 21, Aug. 1953, pp., 1374–1382.

Note on the Infrared Absorption of the Flame of a Simple Double-Base Powder, by Horace F. White, George R. Cowan, David Rotenberg, and Bryce Crawford, Jr., *J. Chem. Phys.*, vol. 21, Sept. 1953, p. 1399.

The Detonation of Pentaery-Thritol Tetranitrate by M. P. Murgai, *J. Chem. Phys.*, vol. 21, Sept. 1953, pp. 1403–1404.

Partial Oxidation of Propane, by C. N. Satterfield, and R. E. Wilson, *Mass. Inst. Tech. Div. of Indust. Cooperation, Hydrogen Peroxide Laboratories Rep.* no. 37, April 1953, 33 pp.

Separation of Hydrogen Peroxide from the Products of the Partial Oxidation of Propane, by C. N. Satterfield, R. E. Wilson, D. O. Cooper, and T. W. Stein. *Mass. Inst. Tech. Div. of Indust. Cooperation, Hydrogen Peroxide Laboratories Rep.* no. 38, April 1953, 29 pp., 4 figs.

Fuels, Propellants, and Materials

Physical Properties of Concentrated Nitric Acid, by W. L. Sibbitt, C. R. St. Clair, T. R. Bump, P. F. Pagerey, J. P. Kern, and D. W. Fyfe, *NACA TN* no. 2970, June 1953, 19 pp.

Studies on Hydrazine. *Tech. Rep.* no. 1, The Photolysis of Ammonia at 1849A in a Flow System, by C. C. McDonald, A. Kahn, and H. E. Gunning, *Illinois Inst. Technology, Dept. Chemistry*, Feb. 1953.

Effect of Prestraining on Recrystallization Temperature and Mechanical Properties of Commercial, Sintered, Wrought Molybdenum, by Kenneth C. Dike and Roger A. Long, *NACA TN* 2973, July 1953, 25 pp.

Investigation of Effects of Grain Size upon Engine Life of Cast AMS 5385, Gas Turbine Blades, by Charles A. Hoffman and Charles A. Gyorgak, *NACA RM* no. E53D06, July 1953, 21 pp.

Behavior of Materials Under Conditions of Thermal Stress, by Samuel S. Manson, *NACA TN* no. July 1953, 105 pp.

Specific Heat of Vitreous Silica, *Rutgers Univ. N. J. Ceramic Res. Sta. Tech. Rep.* no. 7, July 1953, 42 pp.

The Vapour Pressure and Orthobaric Density of Nitrous Oxide by D. Cook, *Trans. Faraday Soc.*, vol. 49, July 1953, pp. 716–723.

Studies and Trends for Aviation Fuels and Lubricants in the United States, by V. Ceccarini, *Aerotecnica*, vol. 33, April 1953, pp. 141–157 (in Italian).

Fuels for Aviation Gas Turbines by P. Givliani, *Rev. Inst. Francaise Pet. Ann. Combustibles Liquides*, vol. 8, June 1953, pp. 282–289 (in French).

Fuels of the Future, Standard Oil Development Co., Linden, N. J., 1953, 48 pp.

Hydrocarbon and Non-Hydrocarbon Derivatives of Cyclopropane, by Vernon A. Slabey, Paul H. Wise, and Louis C. Gibbons, *NACA Rep.* no. 112, 1953, 18 pp.

Investigation of the Effect of Atmosphere Reaction on the Deterioration of Metals at Elevated Temperatures, by O. Cutler Shepard, *Stanford Univ. Div. Mineral Technology. Final Rep.* Navy Contract N6ori154 Task V, July 30, 1953, 24 pp.

Modern Refractory Materials. Aluminum-Chromium "Chromal," by S. Tacvorian, *Recherche Aèron.*, no. 33, May-June 1953, pp. 51–56 (in French).

Physical-Chemical Topics

Contribution to the Theory of Fast Reaction Rates, by R. DeVogelaere, and M. Boudart, *Princeton Univ. James Forrestal Res. Cent. Chemical Kinetics Project Tech. Rep.* no. 3, July 1953, 40 pp., 6 figs.

Kinetics and Mechanisms of the "Water-Gas" Reactions by W. M. Graven and F. J. Long, *Princeton Univ. James Forrestal Res. Cent. Chemical Kinetics Project Tech. Rep.* no. 2, May 1953, 27 pp., 19 figs.

Transport Properties of Gases Obeying a Modified Buckingham (E/p-Six) Potential, by Edward A. Mason, *Wisconsin Univ. Naval Res. Lab. Rep. ONR-1, June* 1953, 64 pp.

Nonequilibrium Thermodynamics, by Newman A. Hall, *J. Appl. Phys.*, vol. 24, July 1953, pp. 819–825.

The Direct Measurement of the Isothermal Joule-Thomson Coefficient for Gases, by A. Charnley, G. L. Isles, and J. R. Townley, *Proc. Roy. Soc.* Series A., vol. 218, no. 1132, June 9, 1953, pp. 133–143.

The Condensation Line of Air and the Heats of Vaporization of Oxygen and Nitrogen, by George T. Furukawa and Robert E. McCoskey *NACA TN* 2969 (formerly *National Bur. Stands. Rep.* 1775), June 1953, 30 pp.

Evaporation of Drops of Liquid, by J. K. Hardy, *Gt. Brit. Aeron. Res. Counc. Rep. Mem.* no. 2805.

Simple General Notations for Systems of Simultaneous Reactions by F. Halla. J. *Phys. Chem.*, vol. 57, June 1953, pp. 599–600.

A Modified Transition State Theory of Reaction Rates, by F. A. Matsen, *Texas Univ. Tech. Rep.*, no. 6, June 1953, 12 pp.

The Quantum Behaviour of Compressed Gases, by H. G. David and S. D. Hamann, *Trans. Faraday Soc.*, vol. 49, part 7, July 1953, pp. 711–716.

Comment on the Temperature Dependence of the Viscosity of Liquids, by Arthur K. Doolittle, *J. Appl. Phys.*, vol. 24, Aug. 1953, p. 1067.

Further Note on the Temperature Dependence of the Viscosity of Liquids, by F. Gutmann, and L. M. Simmons, *J. Appl. Phys.*, vol. 24, Aug. 1953, pp. 1067–1068.

Isotherms of Air Between 0 °C and 75 °C and at Pressures up to 2200 atm., by A. Michels, T. Wassenaar and W. Van Seventer, *Appl. Sci. Res.*, Sec. A, vol. 4, 1953, pp. 52–56.

Luminescence Produced as a Result of Intense Ultrasonic Waves, by Virginia Griffing and Daniele Sette, *Catholic Univ. of America, Tech. Rep.* no. 2, 1953, 15 pp.

Energy Distribution of Luminescence Spectra of Organic Compounds, *Colorado Univ. Dept. of Chemistry, Tech. Rep.* no.1. (1953), 19 pp.

Carbon-Steam Reaction at Low Temperatures, by W. R. Lewis, E. R. Gilliland and Howard Hipkin, *Ind. Engng. Chem.*, vol. 45, Aug. 1953, pp. 1697–1703.

Theory for Irreversible and Constant-Pattern Solid Diffusion, by Theodore Vermeulen, *Ind. Engng. Chem.* vol. 45, Aug., 1953, pp. 1664–1670.

Kinetic Studies with a Fast-Scanning Mass Spectroscope, by Edonard G. Leger and Cyrias Ouellet, *J. Chem. Phys.*, vol. 21, Aug., 1953, pp. 1664–1670.

Bond Energies and Bond Distances of Hydrocarbons, by George Glockler, *J. Chem. Phys.*, vol. 21, July 1953, pp. 1242–1248.

Theory of the Rotational Spectra of Allene-Type Molecules, by Masataka Mizushina, *J. Chem. Phys.*, vol. 21, July 1953, pp. 1222–1224.

On Variational Principles for Irreversible Processes by Philip Rosen, *J. Chem. Phys.*, vol. 21, July 1953, pp. 1220–1221.

Free Radicals by Mass Spectrometry, IV. The Rate of Combination of Methyl Radicals, by K. U. Ingold and F. P. Lossing, *J. Chem. Phys.*, vol. 21, July 1953, pp. 1135–1144.

Application of a Fast-Scanning Infrared Spectrometer to a Kinetic Study of the

NO_2—C_2H_5OH Reaction, by Richard E. Nightingale, George R. Cowan, and Bryce Crawford, Jr., *J. Chem. Phys.*, vol. 21, Aug. 1953, pp. 1398–1399.

Some New Data on the NO—N_2O_5 Reaction, by George R. Cowan, David Rotenberg, Arthur Downie, Bryce Crawford, Jr., and R. A. Ogg, Jr., *J. Chem. Phys.*, vol. 21, Aug. 1953, pp. 1397–1398.

Kinetics of the Interaction of Atomic Hydrogen with Olefines, VI. An Extension of the Method and Application to Substituted Olefines and Aromatics, by P. E. M. Allen, H. W. Melville, and J. C. Robb, *Proc. Roy. Soc., Ser. A*, vol. 218, July 7, 1953, pp. 311–327.

Studies in Energy Transfer. III. The Effect of Added Gases on the Decomposition of Cyclobutane, by H. O. Pritchard, R. G. Sowden, and A. F. Trotman-Dickenson, *Proc. Roy. Soc., Ser. A*, vol. 218, July 7, 1953, pp. 416–421.

Influence of Nitric Oxide on the Two Opposing Reactions of the Equilibrium $C_2H_6 \rightleftharpoons C_2H_4 + H_2$, by B. C. Spall, F. J. Stubbs, and Cyril Hinshelwood, *Proc. Roy. Soc., Ser. A*, vol. 218, July 23, 1952, pp. 439–449.

Electrical Properties of Titanium Dioxide Semi-Conductors, by Robert G. Breckenridge and William R. Hosler, *Phys. Rev.*, vol. 91, Aug. 15, 1953, pp. 793–802.

Diffusion in Binary Systems, by Stephen Prager, *J. Chem. Phys.*, vol. 21, Aug. 1953, pp. 1344–1347.

Thermal Diffusion in Binary Systems, by Stephen Prager and Henry Eyring, *J. Chem. Phys.*, vol. 21, Aug. 1953, pp. 1347–1350.

The Rate Constant of the Reaction Between Hydrogen Peroxide and Ferrous Ions, by William Taylor and Joseph Weiss, *J. Chem. Phys.*, vol. 21, Sept. 1953, pp. 1419–1420.

Molecular Sound Velocity and Molecular Compressibility of Liquid Mixtures, by Otohiko Nomoto, *J. Phys. Soc. of Japan*, vol. 8, July-Aug. 1953, pp. 553–560.

The Irreversible Formation of Methane in the System Ethane-Ethylene-Hydrogen, by C. J. Danby, B. C. Spall, F. J. Stubbs, and Cyril Hinshelwood, *Proc. Roy. Soc., Ser. A*, vol. 218, July 23, 1953, pp. 450–464.

Virial Coefficients and Models of Molecules in Gases, by Taro Kihara, *Wisconsin Univ. Naval Res. Lab. Rep.* OOR-7, June 1953, 49 pp.

The Direct Measurement of the Isothermal Joule-Thomson Coefficient for Gases, by A. Charnley, G. L. Isles, and J. R. Townley. *Proc. Roy. Soc.*, vol. 218A, no. 1132, June 9, 1953, pp. 133–143.

Physical Properties of Concentrated Nitric Acid, by W. L. Sibbitt, C. R. St. Clair, T. R. Bump, P. F. Pagerey, J. P. Kern, and D. W. Fyfe, *NACA TN* no. 2970. June 1953, 19 pp.

The Condensation Line of Air and the Heats of Vaporization of Oxygen and Nitrogen, by George T. Furukawa and Robert E. McCoskey, *NACA TN* 2969 (formerly *Nat. Bur. Stand. Rep.* 1775), June 1953, 30 pp.

Instrumentation and Experimental Techniques

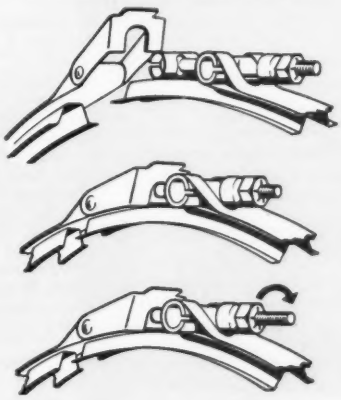
Continuous Recording of Pressures for Supersonic Wind-Tunnel Calibration, by L. P. Gieseler, *NAVORD Rep.* no. 2744, Jan. 1953, 14 pp., 7 figs.

Measurement of Timed Spark Shadowgraphs of Shock Velocities in the Shock

Speed up production

with MARMAN *Quick* Coupler Latches

Exclusive Swivel Action...No loose parts...



Quick TO OPEN

Quick TO CLOSE

Quick TO TIGHTEN

Positive Lock Under All Conditions

Not affected by heat, cold, vibration or stress

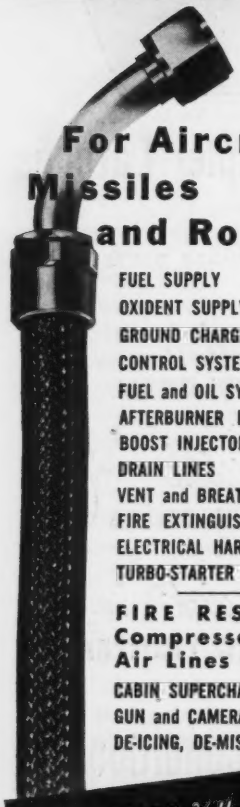
These time-saving features **are available on the standard line of Marman precision couplings—easy to specify—fast on delivery**

For catalog or information, write Dept. S-5.

MARMAN PRODUCTS CO., Inc.
11214 EXPOSITION BLVD.
LOS ANGELES 64, CALIF.

3-429

AVICA
LIGHTWEIGHT
CORROSION RESISTANT
Stainless Steel
FLEXIBLE
TUBE ASSEMBLIES



For Aircraft Missiles and Rockets

FUEL SUPPLY
OXIDENT SUPPLY
GROUND CHARGING
CONTROL SYSTEM UNITS
FUEL and OIL SYSTEMS
AFTERBURNER INSTALLATIONS
BOOST INJECTOR SYSTEMS
DRAIN LINES
VENT and BREATHER LINES
FIRE EXTINGUISHER SYSTEMS
ELECTRICAL HARNESS
TURBO-STARTER ASSEMBLIES

FIRE RESISTANT
Compressor Bleed
Air Lines for:—
CABIN SUPERCHARGING
GUN and CAMERA HEAT
DE-ICING, DE-MISTING

SIZE RANGE 3/16" —4" I.D.
MECHANICALLY Applied
Reusable END FITTINGS

AVICA is always ready to discuss special applications with customers and to develop hose assemblies to deal with unusual operating conditions.

WRITE TO SPECIAL PROJECTS DEPT. A. W. FOR FURTHER INFORMATION

AVICA CORPORATION
P. O. BOX 1090
PORTSMOUTH, RHODE ISLAND
TEL. PORTSMOUTH 479

WEST COAST REPRESENTATIVE
AIRSUPPLY COMPANY, 9815 WILSHIRE, BLVD.
BEVERLY HILLS, CALIFORNIA

Tube, by Herbert L. Hoover, *Lehigh Univ. Inst. of Res. Tech. Rep.* no. 3, July 1953, 24 pp.

A Positive Method for Locating the Zero-Order Fringe in Interferograms Produced with Filtered Light, by Henry W. Wedaa and Edward W. Price, *J. Aero. Sci.*, vol. 20, July 1953, p. 506.

Carbon-Composition Thermometers at Very Low Temperatures, by J. R. Clement, E. H. Quinnell, M. C. Steele, R. A. Hein, and R. L. Dolecek, *Rev. Sci. Instrum.* vol. 24, July 1953, pp. 545–546.

Analogies for Hydraulic and Electric Drives in Servomechanisms, by Yaohan Chu and L. A. Gould, *Trans. ASME*, vol. 75, July 1953, pp. 851–857.

Apparatus for Rapid Determination of Liquid-Liquid and Liquid-Solid Equilibria Involving Corrosive Material, by W. Davis, Jr., L. L. Conley, and G. P. Rutledge, *Rev. Sci. Inst.*, vol. 24, June 1953, pp. 431–432.

A New Method for Continuous Viscosity Measurement, by Wilfred Roth and Stanley R. Rich, *J. Appl. Phys.*, vol. 24, July 1953, pp. 940–950.

The Gyroscope in Aeronautics, by A. H. Yates, *Flight*, vol. 64, July 17, 1953, pp. 78–79.

Membrane Apparatus for Analogue Experiments, by Paul J. Schneider and Ali B. Cambel, *Rev. Sci. Instrum.*, vol. 24, July 1953, pp. 513–514.

A Rapid-Scanning Auroral Spectrometer by D. M. Hunten, *Can. J. Phys.*, vol. 31, July 1953, pp. 681–690.

Procedure for Measuring Liquid Water Content and Droplet Sizes in Supercooled Clouds by Rotating Multicylinder Method, by William Lewis, Porter J. Perkins, Rinaldo J. Brun, and Paul T. Hacker, *NACA RM* no. E53D23, June 1953, 48 pp.

Calibration of Strain Gage Installations in Aircraft Structures for the Measurement of Flight Loads, by T. H. Skopinski, William S. Aiken, Jr., and Wilber B. Huston, *NACA TN* 2993 (formerly *NACA RM* L52G31), Aug. 1953, 70 pp.

Interpolation Servomechanisms, Final Report No. UA-624-P-1. *Cornell Aeron. Lab., Inc.*, 1953, 23 pp.

Schlieren Photographs of Sound Fields, by Daniel S. Schwartz and Anthony L. Russo, *J. Appl. Phys.*, vol. 24, Aug. 1953, pp. 1061–1062.

Rocket Motor Testing by D. Hurden, *J. Brit. Interplan. Soc.*, vol. 12, May 1953, pp. 101–113.

A Magnetic Amplifier for Synchros, by R. L. Van Allen, *Naval Res. Lab. Rep.* no. 4174, June 1953, 16 pp.

A Positive Method for Locating the Zero-Order Fringe in Interferograms Produced with Filtered Light, by Henry W. Wedaa and Edward W. Price, *J. Aero. Sci.*, vol. 20, July 1953, p. 506.

The Relation Between Open-Loop Stability and Frequency Response Diagrams for Linear Systems, by Daniel O. Dommasch, *J. Aero. Sci.*, vol. 20, July 1953, pp. 506–509.

Application of a Three Phase Motor to Servomechanisms, by John E. Hart, *Naval Res. Lab. Rep.* no. 4168, May 1953, 32 pp.

Analogies for Hydraulic and Electric Drives in Servomechanisms, by Yaohan Chu and L. A. Gould, *Trans. ASME*, vol. 25, July 1953, pp. 851–857.

Space Flight

Landing on Airless Planets, by R. A. Smith, *J. Brit. Interplan. Soc.*, vol. 12, July 1953, pp. 149–155.

The Food and Atmosphere Control Problem in Space Vessels, II. The Use of Algae for Food and Atmosphere Control, by Norman J. Bowman, *J. Brit. Interplan. Soc.*, vol. 12, July 1953, pp. 159–167.

Space Physiology, by C. R. Armstrong, *J. Brit. Interplan. Soc.*, vol. 12, July 1953, pp. 172–175.

The Fundamentals of Astronautics, by G. A. Crocco, *Aerotecnica*, vol. 33, April 1953, pp. 135–140 (in Italian).

Space Flight Congress, by A. E. Slater, *Aeroplane*, vol. 85, Aug. 21, 1953, pp. 230–231.

Planetary Engineering, by E. F. Hope-Jones, *J. Brit. Interplan. Soc.*, vol. 12, July 1953, pp. 155–159.

Terrestrial Flight, Ballistics, and Vehicle Design

Typical Problems in the Design of Liquid Propellant Missiles, by Vittorio Sivori (in Italian), *Aerotecnica*, vol. 33, Feb. 1953, pp. 118–122.

Recent Developments in Foreign Rockets and Guided Missiles, by Norman J. Bowman, *J. Space Flight*, vol. 5, June 1953, pp. 1–4.

Rockets Breathe New Life into Autogyro, by David A. Anderton, *Aviation Week*, vol. 58, June 22, 1953, pp. 28–30.

Astrophysics, Aerophysics, and Atomic Physics

Effect of Radar Sensitivity on Meteor Echo Durations, by D. W. R. McKinley, *Can. J. Phys.*, vol. 31, July 1953, pp. 758–767.

New Techniques in Radio Astronomy, by John D. Kraus and Edward Ksiazek, *Electronics*, vol. 26, Sept. 1953, pp. 148–152.

Phototubes Observe Total Solar Eclipse, by Francis J. Heyden, *Electronics*, vol. 26, Sept. 1953, pp. 176–179.

Experimental Studies of the Motions of Slow Electrons in Air with Application to the Ionosphere, by R. W. Crompton, L. G. H. Huxley, and D. J. Sutton, *Proc. Roy. Soc., Ser. A*, vol. 218, July 23, 1953, pp. 507–519.

Electron Excitation of Forbidden Lines Occurring in Gaseous Nebulae, by M. J. Seaton, *Proc. Roy. Soc., Ser. A.* vol. 218, July 7, 1953, pp. 400–416.

Energy Production in Stars, by E. E. Salpeter, *Ann. Rev. Nuclear Science*, vol. 2, 1953, pp. 41–62.

Origin and Propagation of Cosmic Rays, by Ludwig Biermann, *Ann. Rev. Nuclear Science*, vol. 2, 1953, pp. 335–364.

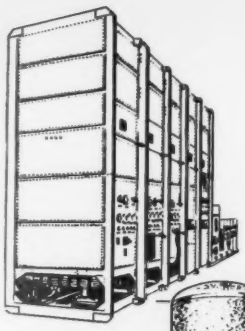
High Altitude Measurements of the Intensity of Cosmic Radiation at Magnetic Latitudes 3°N and 19°N, by A. S. Rao, V. K. Balasubrahmanyam, G. S. Gokhale, and A. W. Pereira, *Phys. Rev.*, vol. 91, Aug. 1, 1953, pp. 764–765.

Atomic Aircraft, How Much?, by Richard J. Moffett, *Can. Aviation*, vol. 26, Aug. 1953, pp. 23, 37–38, 40, 42.

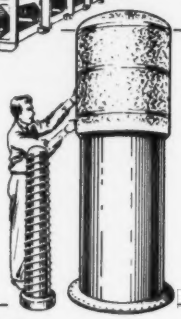
Atomic Aircraft, When and Why, by Norman S. Currey, *Can. Aviotion*, vol. 26, Aug. 1953, pp. 22, 34, 37.

Bubble Chamber Tracks of Penetrating Cosmic-Ray Particles, by Donald A. Glaser, *Phys. Rev.*, vol. 91, Aug. 1 ,1953, pp. 762–763.

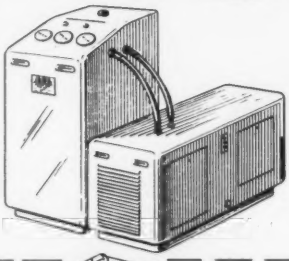
Anisotropic Distribution of Secondaries in Extreme Energy Cosmic-Ray Stars, by Frank C. Roesler and C. B. A. McCusker, *Phys. Rev.*, vol. 91, Aug. 1, 1953, pp. 690–694.



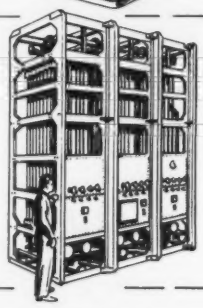
LIQUID OXYGEN GENERATORS
An air-transportable plant for the separation of high-purity oxygen from the air.



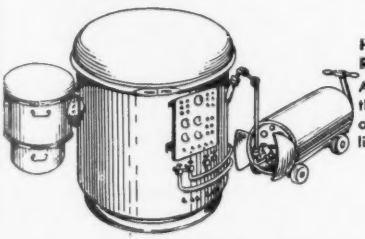
LIQUEFIED-GAS STORAGE CONTAINERS
Containers for the storage or transport of liquefied gases such as liquid hydrogen or oxygen.



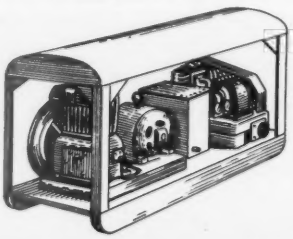
LIQUID OXYGEN PUMPS
A compact, noncontaminating unit for supplying high-pressure oxygen gas from low-pressure liquid oxygen storage containers.



HEAT EXCHANGERS
A gas-to-gas heat exchanger with exceptional characteristics.



HELIUM REFRIGERATORS
A practical system for the prevention of evaporation loss in stored liquefied gases.



AIR COOLERS
A unit which provides cooled, compressed air which is free of dirt, oil, or entrained water.

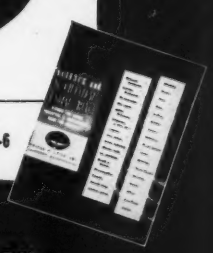
UNIQUE
PROTOTYPE DEVELOPMENT

Arthur D. Little, Inc. has blended its scientific and engineering skills in the Mechanical Division to provide industry with a unique service . . .

Scientists in the fields of chemistry, physics, metallurgy, mathematics, biology, electronics and technical economics regularly engaged in diversified research and development projects join with our engineers in prototype development of equipment requiring a high level of engineering skills.

Our staff is experienced in interpreting the ideas of industry and following through with the perfection of specialized equipment.

ADL
Mechanical Division



WRITE FOR BROCHURE RJ28-6

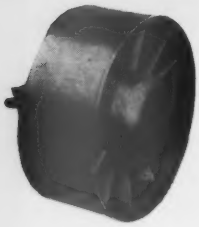
MECHANICAL DIVISION
Arthur D. Little, Inc.
30 MEMORIAL DRIVE — CAMBRIDGE 42, MASS.
Offices in New York, St. Louis, and Mexico City

CREATIVE TECHNOLOGY SINCE 1886

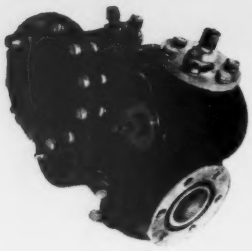
THERMODYNAMICS • HEAT TRANSFER • REFRIGERATION TO MINUS 456°F • GAS LIQUEFACTION • VACUUM ENGINEERING • ELECTROMAGNETISM • MECHANICAL DESIGN • VIBRATION

Flight Tested!

AIR TURBINE *Accessory Drives* and THRUST *CONTROLS* of advanced design

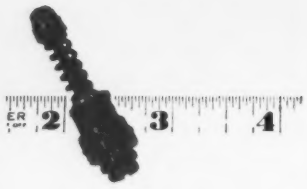


Ram air turbine driven unit for *emergency* aircraft power.

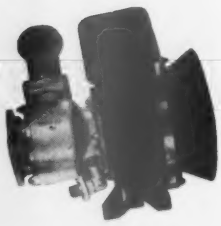


Supersonic ramjet engine power control.

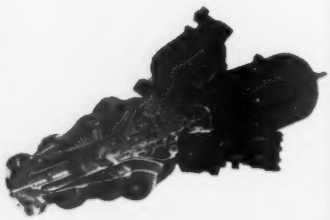
40 HP turbine driven unit, supplying electricity and hydraulic power.



Advanced design fuel nozzle for ramjets and afterburners.

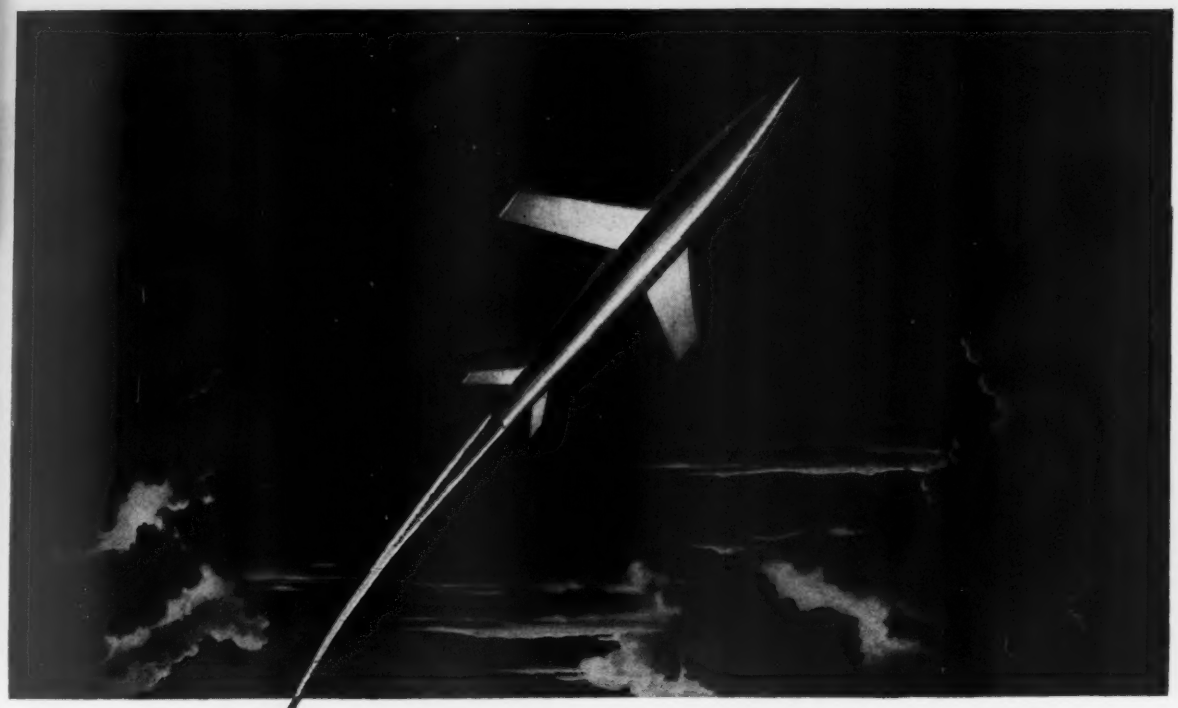


Air turbine driven fuel pump.



50 HP turbine driven unit, supplying electricity and hydraulic power.

marquardt AIRCRAFT CO.



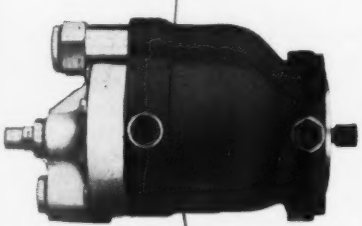
OUTWARD BOUND · · · Controlled All the Way by—

STRATOPOWER
HYDRAULIC PUMPS

To the layman, the pinpointing of a destination for a guided missile is still pure magic. But to the engineer, the control and guidance of rockets and guided missiles in flight simply means another application for STRATOPOWER quality and advanced design.

The unerring performance of STRATOPOWER Pumps has been demonstrated times without number down through the years. At sea level and at heights still to be achieved these perfect examples of precision engineering provide the fluid power that positively answers the question of weight vs. horsepower as well as the equally important requirement of long-lived dependability.

In the designing of any high pressure hydraulic circuit there are definite advantages in STRATOPOWER Pumps. Whether yours is a problem for constant or variable delivery, high or low temperature operation, capacities from .25 to 30 gpm, STRATOPOWER will provide the pump to 3000 psi that will resolve that problem NOW!



The 66W Series of STRATOPOWER Hydraulic Pumps embrace a range of models from 2 to 10 gpm at 1500 rpm with continuous pressures to 3000 psi. Designed for maximum continuous speed of 3750 rpm . . . intermittent speeds to 4500 rpm.

Write ***Get the full story on STRATOPOWER constant and variable delivery Pumps for your hydraulic circuits.***

WATERTOWN DIVISION
THE NEW YORK AIR BRAKE COMPANY
STARBUCK AVENUE • WATERTOWN • N. Y.

WATERTOWN DIVISION
THE NEW YORK AIR BRAKE COMPANY
730 Starbuck Avenue • Watertown, N. Y.

Kindly send me information on STRATOPOWER Hydraulic Pumps

☐ Constant delivery ☐ Variable delivery

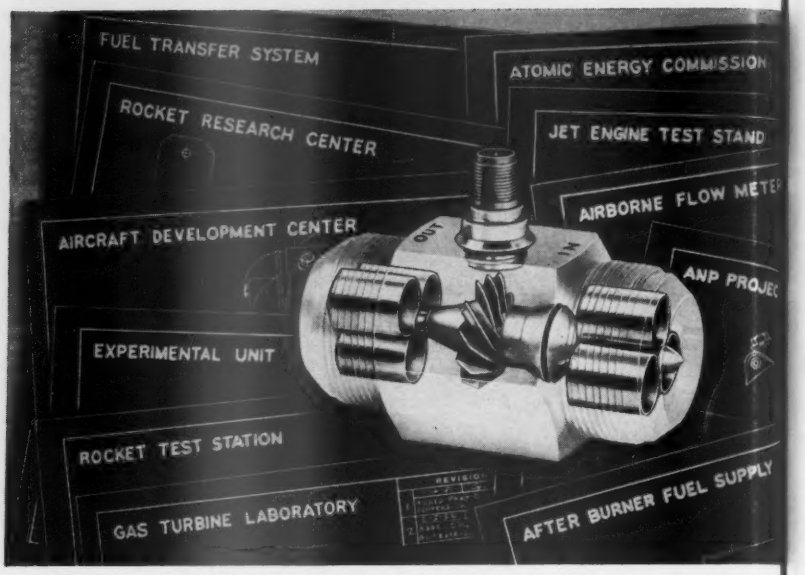
Name_____

Company_____

Address_____

City_____ Zone___ State_____

Potter Flowmeter First in Rocket Research



This unique flow sensing element, able to withstand high pressures and high or low temperatures, has brought a new standard of accuracy to flow measurement in test and research work. Resistant to highly corrosive liquids, it is also being used in most jet and rocket test projects.

ROCKET RESEARCH INSTALLATIONS throughout the United States and Canada have adopted the Potter Flowmeter as their standard for accuracy. Simplicity, stability, dependability, and ruggedness, combined with an ability to operate in either the vertical or horizontal position are only a few of the features that have caused leading aircraft research organizations to specify Potter for all new test installations.

ROCKET PROPELLANTS are measured easily and accurately with the Potter unit, which withstands high or low temperatures, high pressures and strong acids. Liquid oxygen, fuming nitric acid, hydrogen peroxide, liquid nitrogen and other liquids considered "hard to handle" are being measured by Potter systems with safety and precision.

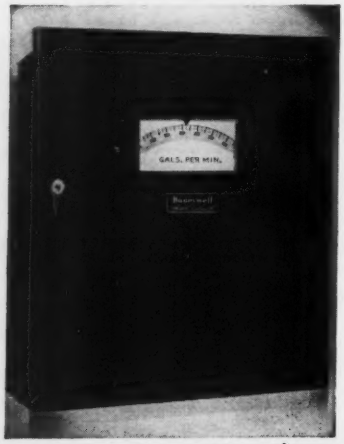
ENGINE TEST CELLS have standardized on Potter for *safety* as well as ease of installation, wide rangeability and accuracy. The Potter sensing element is inherently explosion proof, unbreakable, and has no gaskets, pressure seals or stuffing box. The electrical output is too low to cause a spark under any possible conditions.

ACCURATE READING of flow rates has been made possible through the development of instruments whose precision approaches the Potter sensing element. The Potter-Brown Precision Flow Indicator has a scale with a graduated length of 28 inches, marked with as many as 500 divisions. Since the flowmeter is linear, legibility is excellent at high or low flow rates. If necessary, the effective scale length can be *doubled* by using a dual scale.

RECORDING FLOW on either a circular or strip chart provides permanent test records. The Potter-Brown Strip Chart Recorder, having a graduated chart width of ten inches and available in a variety of chart speeds provides accuracy, convenience, and ease of reading.

TOP SECRET labels are on many of the projects in which Potter equipment is being used; but most users are able to give reports on the operation of their Potter Flowmeters. If you are measuring flow, and you're not already using the Potter Flowmeter, ask your associates and acquaintances in the industry about this new method of measuring flow. Their experience may help you to solve your flow measuring problems.

FOR FURTHER INFORMATION. State whether you are interested in indicating, recording or controlling flow rate. Watch for important news on the new Potter Airborne Flowmeters for measuring rate and total flow. Write to:

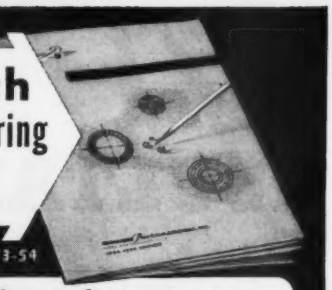


Twenty-eight inch rotating disc scale gives Potter-Brown Flow Indicator superior legibility.

POTTER AERONAUTICAL COMPANY

87 Academy St., Newark 2, N. J. **Phone Mitchell 2-5525**

. . . which
MPB ball bearing
do you need?



new MPB catalog 53-54

Most complete information ever offered on miniature ball bearings

- Complete specifications – 140 different types and sizes
- Bearings from 1/10" to 3/8" o.d. shown in actual sizes
- Speed-load charts with conversion factor
- Lubrication–government specifications, commercial sources
- Recommended shaft, housing fits; shaft, shoulder data
- Radial and axial clearance graphs
- Typical methods of using miniature ball bearings

For the designer of precision mechanisms this new 20 page MPB catalog offers practical solutions of problems involving miniaturization. MPB has compiled for you the most complete and detailed information ever offered on this subject. Request the new MPB catalog 53-54 on your letterhead . . . it may help develop a new product idea for you.

MPB
Miniature Precision Bearings
Incorporated Keene, New Hampshire

Reynolds Electrical and Engineering Co., Inc.

Electrical and Construction Engineers

EL PASO
HOUSTON
ALBUQUERQUE
SANTA FE
LAS VEGAS

AN ORGANIZATION OF ELECTRICAL ENGINEERS TRAINED TO THE INTRICATE NEEDS OF THE CONSTRUCTION INDUSTRY

WANTED

PHYSICISTS

MS or PhD PREFERRED

Experienced in
HEAT ENGINES and THERMODYNAMICS. HIGH PRESSURE experience also valuable or with knowledge of:
FLUID FLOW . . . HIGH SPEED PHOTOGRAPHY . . .
HIGH TEMPERATURE MEASUREMENT

ANALYTICAL CHEMISTS

Graduate degree desirable. To do quantitative analysis in inorganic and organic chemistry; and non-routine analysis including some research.

INFRA-RED SPECTROSCOPIST

Knowledge or experience in analytical procedures of organometallics, hydrocarbons and inorganic compounds.

CHEMIST OR CHEMICAL ENGINEER

Experienced in high pressure flow reactions.

LET'S TALK FACTS ABOUT ROCKET POWER

Rocket power is no longer "the coming thing." It's here . . .

Nowhere is the fact more apparent than at Reaction Motors, pioneer and leading research and development organization whose engines have powered most of the record-breaking flights. A comparatively small company, RMI provides an ideal stimulating environment for young physicists and chemists with imaginative, inquisitive minds. Located in the Lakeland region of New Jersey, renowned vacation playground, it offers ideal year-round living conditions, with ample housing.

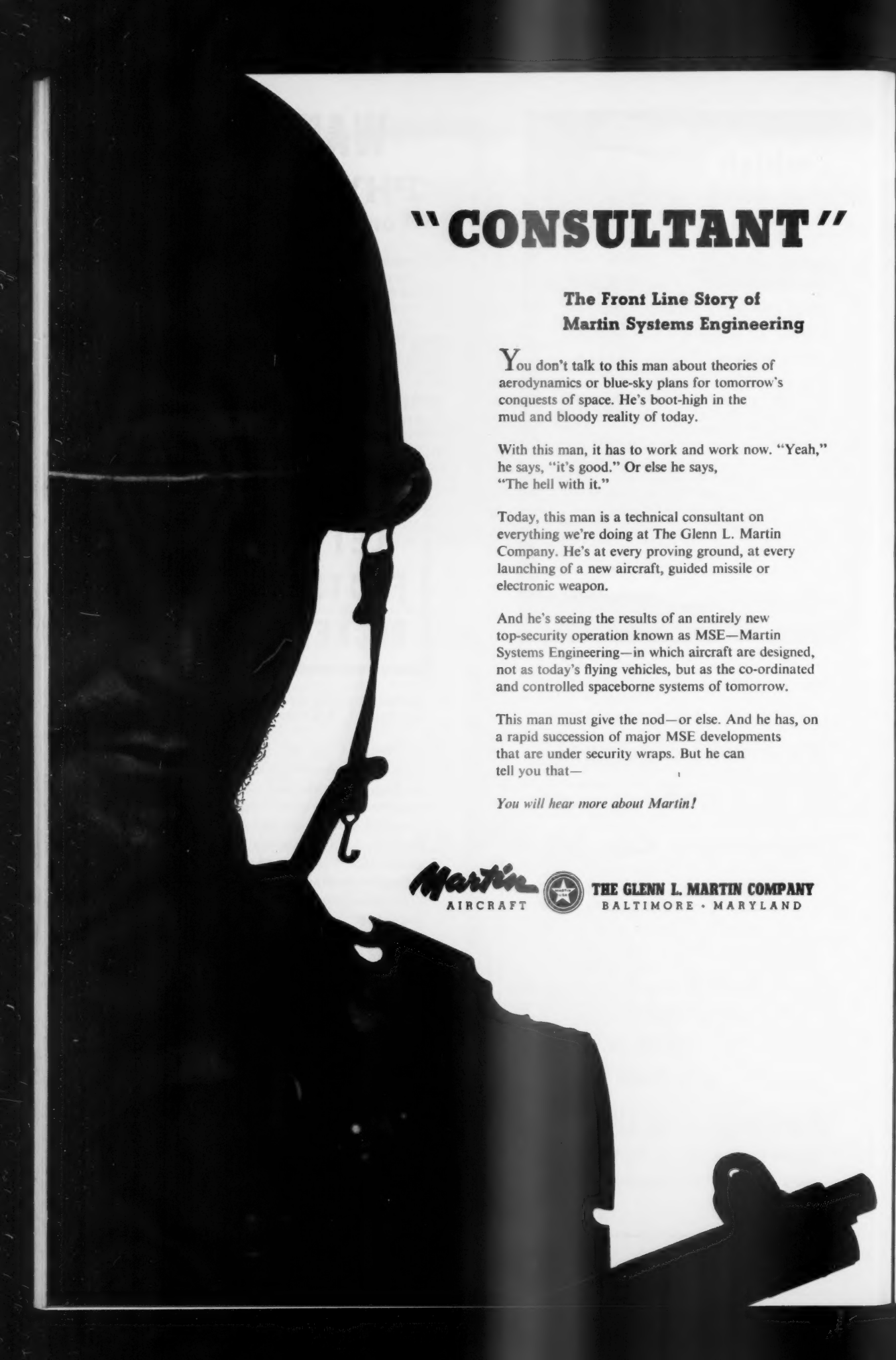
IF YOU'RE INTERESTED . . .
IF YOU QUALIFY . . .
Send complete resume and salary requirements to:

EMPLOYMENT MANAGER

REACTION MOTORS, INC.

Foremost Designers & Developers of Rocket Power

ROCKAWAY, NEW JERSEY
LESS THAN 1 HOUR FROM TIMES SQUARE



"CONSULTANT"

The Front Line Story of Martin Systems Engineering

You don't talk to this man about theories of aerodynamics or blue-sky plans for tomorrow's conquests of space. He's boot-high in the mud and bloody reality of today.

With this man, it has to work and work now. "Yeah," he says, "it's good." Or else he says, "The hell with it."

Today, this man is a technical consultant on everything we're doing at The Glenn L. Martin Company. He's at every proving ground, at every launching of a new aircraft, guided missile or electronic weapon.

And he's seeing the results of an entirely new top-security operation known as MSE—Martin Systems Engineering—in which aircraft are designed, not as today's flying vehicles, but as the co-ordinated and controlled spaceborne systems of tomorrow.

This man must give the nod—or else. And he has, on a rapid succession of major MSE developments that are under security wraps. But he can tell you that—

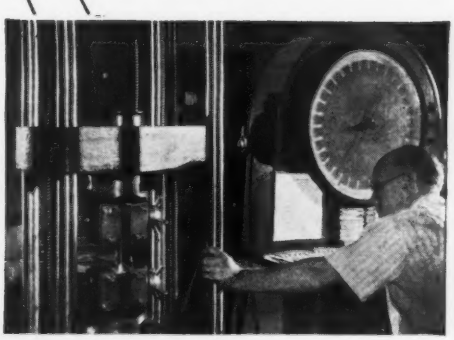
You will hear more about Martin!

Martin AIRCRAFT

THE GLENN L. MARTIN COMPANY
BALTIMORE · MARYLAND

more performance per pound with new alloy aluminum extrusions

Harvey Aluminum has developed a new aluminum alloy having typical ultimate mechanical properties of 100,000 PSI. Guaranteed properties are above those of any aluminum extrusion alloy yet developed. Long range research at Harvey has produced the best possible alloys for high stress structural application. Mechanical properties of this new alloy, HZM100, are higher than any other aluminum alloy. Another feature of HZM100 of particular interest to structural engineers is its satisfactory stress corrosion resistance.



Proving tensile strength of test specimens in all aluminum alloys is part of the quality control carried on continuously at Harvey Aluminum.

MAKING THE MOST OF ALUMINUM . . FOR INDUSTRY

HARVEY Aluminum

An independent facility producing special extrusions, pressure forgings, bar stock, forging stock, tubing, aluminum screw machine products, and related mill products.

HARVEY ALUMINUM SALES, INC.
TORRANCE • LOS ANGELES, CALIFORNIA
BRANCH OFFICES IN PRINCIPAL CITIES

"MONOBALL" ®

SELF-ALIGNING BEARINGS

CHARACTERISTICS

ANALYSIS	RECOMMENDED USE
1. Stainless Steel Ball & Race	1. For types operating under high temp. (800—1200 degrees F.)
2. Chrome Molly Steel Ball & Race	2. For types operating under high radial ultimate loads (3000—893,000 lbs.)
3. Bronze Race & Chrome Molly Steel Ball	3. For types operating under normal loads with minimum friction requirements.

Thousands in use. Backed by years of service life. Wide variety of Plain Types in bore sizes 3/16" to 6" Dia. Rod end types in similar size range with externally or internally threaded shanks. Our Field Engineers welcome an opportunity of studying individual requirements and prescribing a type or types which will serve under your demanding conditions. Southwest can design special types to fit individual specifications. As a result of thorough study of different operating conditions, various steel alloys have been used to meet specific needs. New types have been developed, many as a result of specific requirements of our customers. Write or wire now for revised Engineering Manual describing complete line of "Monoball" Self-aligning Bearings and Rod Ends. Please address Dept. R-53.

SOUTHWEST PRODUCTS CO.
1705 S. Mountain Ave. Duarte, Calif.

Precision

ORIFICES

We are the leading manufacturer of precision orifices in the world. We manufacture Spinnerettes used to produce synthetic fibers. Each Spinnerette requires from 2 to 25,000 precision orifices with tolerances of plus or minus 0.00004 inches. Let this background help your production of precision fuel injector nozzles. Write or call Department O for information.

BAKER & CO., INC.
113 ASTOR STREET, NEWARK 5, NEW JERSEY

SCINTILLA MAGNETO DIVISION

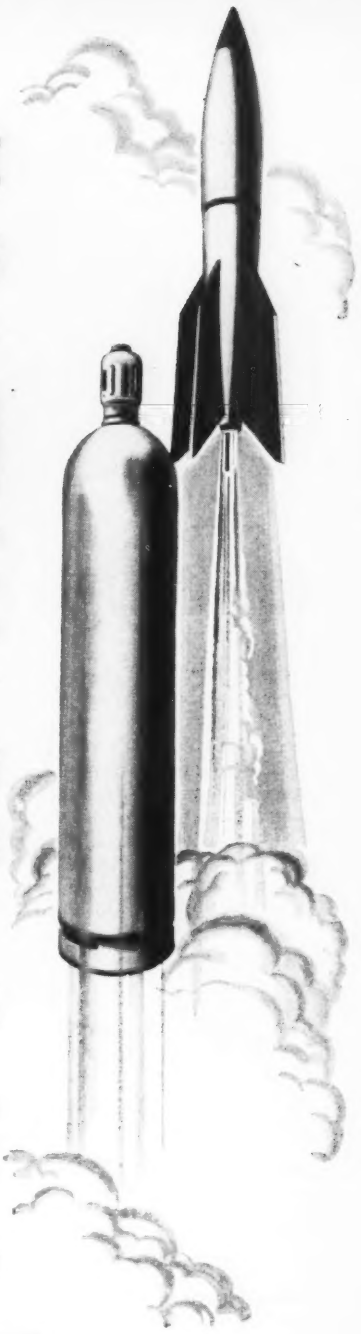
BENDIX AVIATION CORPORATION
Sidney, New York

Manufacturers of Ignition Systems for Jet, Turbine, Piston Power Plants, and Rocket Motors; Electrical Connectors; Ignition Analyzers, Moldings and other Components and Accessories.

As an Oxidant in Liquid Propellants

NITROGEN TETROXIDE

offers outstanding advantages to designers of rocket motors



HIGH SPECIFIC IMPULSE: Nitrogen Tetroxide exceeds many other well-known oxidizers in pounds of thrust developed per pound of fuel consumed per second.

EASY TO HANDLE: Nitrogen Tetroxide may be shipped, piped and stored in ordinary carbon steel equipment. It possesses high chemical stability, high density, low freezing point, and a reasonably low vapor pressure.

Nitrogen Tetroxide is available at low cost in 125-pound I.C.C. approved steel cylinders and 1-ton containers.

Address your inquiry to the Product Development Department

NITROGEN DIVISION ALLIED CHEMICAL

Nitrogen Division

ALLIED CHEMICAL & DYE CORPORATION

40 RECTOR STREET, NEW YORK 6, N. Y.

Technical service and development on Nitrogen Tetroxide—formerly handled by the Product Development Department, Solvay Process Division—are now handled by Nitrogen Division, Allied Chemical & Dye Corporation.

YOU helped create this superior Recording Oscillograph

NEW Type 5-119

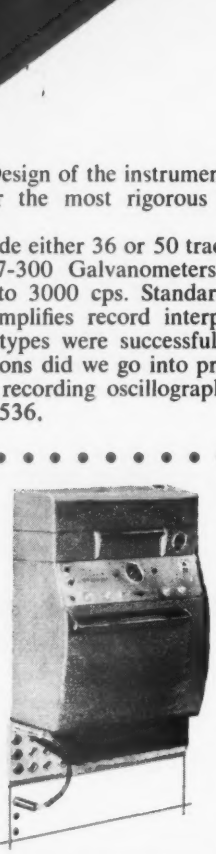
CONSOLIDATED's new Type 5-119 Oscillograph was designed and built to customer specifications. Many major users were interviewed by our design engineers to determine the features desired in a "perfect" instrument. Foremost demand was for great dependability. Second need indicated was for high trace capacity.

Dependability is achieved by extensive warning and test circuits and by reserve lamps which assure continuous recording in the event of lamp burn-out. Indicators warn immediately of any condition which could cause data loss, while additional circuits permit quick testing of the warning system. Design of the instrument assures reliable operation under the most rigorous environmental conditions.

Standard models provide either 36 or 50 traces. Consolidated's new Series 7-300 Galvanometers provide frequency response flat to 3000 cps. Standard record width of 12″ greatly simplifies record interpretation. Only after several prototypes were successfully tested under actual field conditions did we go into production on the 5-119—the new recording oscillograph leader. Write for Bulletin CEC 1536.

SPECIFICATIONS

TRACE CAPACITY	36 or 50-trace models available
TRACE IDENTIFICATION	repeated, sequential trace breaks
RECORD WIDTH	12″ standard; narrower widths adaptable
RECORD MAGAZINE	removable, integral type; holds 250′ paper or film
RECORD SPEEDS	0.10 to 100 inches per sec. through quick-change gears, instantaneous switch-actuated, 10:1 speed jump
SCANNING SYSTEM	ground-glass screen and adjustable motor-driven polygon mirror; timing lines show on viewing screen
REMOTE OPERATION	accessory control unit with all essential controls & indicators
TIMING PROVISIONS	0.10 and/or 0.01 sec. lines photographed across record
EVENT NUMBERING	high-speed flash system operates as rapidly as one number per sec.
POWER REQUIREMENTS	115 volt, 60 cycle and 26 volt d-c models
INPUT PROVISIONS	all connectors on one rear deck; individual galvanometer plugs
CONTROL PANEL	all controls and indicators on single panel
ACCESSIBILITY	all adjustments can be made from one surface



The 5-119 can be panel mounted vertically with special shockmounts.

Consolidated Engineering

CORPORATION

300 North Sierra Madre Villa, Pasadena 15, California

Sales and Service through **CEC INSTRUMENTS, INC.,** *a subsidiary with offices in: Pasadena, New York, Chicago, Washington, D. C., Philadelphia, Dallas.*

analytical instruments for science and industry

Recording Oscillographs

The Type 5-119 is the newest of 7 Consolidated Oscillographs ranging from 9 to 50 channel capacity. These versatile instruments simultaneously record any physical phenomena that can be transformed into electrical impulses. All measurements are obtained in clear, permanent form during the same operational cycle for future detailed analysis.

Index to Advertisers

AEROJET-GENERAL CORP. ..... outside back cover

ALLIED CHEMICAL & DYE CORP.
NITROGEN DIV. 403
Albert Sidney Noble Advertising, New York, N. Y.

AUDIO PRODUCTS CORP. 340
McCarty Co., Los Angeles, Calif.

AVICA CORP. 394
Knight & Gilbert, Inc., Providence, R. I.

BAKER & CO. 402

BARROWS PORCELAIN ENAMEL CO. 386
Perry Brown, Inc., Cincinnati, Ohio

BECKMAN INSTRUMENTS, INC.
BERKELEY SCIENTIFIC DIV. 405

BELL AIRCRAFT CORP. 342
Cometock & Co., Buffalo, N. Y.

BENDIX AVIATION CORP.
SCINTILLA MAGNETO DIV. 402
The Shaw Co., Los Angeles, Calif.

CONSOLIDATED ENGINEERING CORP. 404
Hixson & Jorgensen, Inc., Los Angeles, Calif.

CURRAN ENGINEERING COMPANY 382

DOUGLAS AIRCRAFT CO. 345
J. Walter Thompson Co., Los Angeles, Calif.

EXCELCO DEVELOPMENTS, INC. 407

FAIRCHILD ENGINE AND AIRPLANE CORP.
AIRCRAFT DIV. 339
STRATOS DIV. 346
Buchanan & Co., Inc, New York, N. Y.

FORD INSTRUMENT CO. 338
G. M. Basford Co., New York, N. Y.

GRAND CENTRAL AIRCRAFT 348

GYROMECHANISMS, INC. ..... inside back cover
Corydon M. Johnson Co. Inc., Bethpage, N. Y.

HARVEY 401
Hixson & Jorgensen, Inc., Los Angeles, Calif.

HUFFORD MACHINE WORKS, INC. 341
Clyde D. Graham, Los Angeles, Calif.

INDUSTRIAL SOUND CONTROL 376
William Schaller Co., Hartford, Conn.

LAVOIE LABORATORIES, INC. 408
The Picard Advertising Co., New York, N. Y.

LITTLE, ARTHUR D., INC. 395
Larcom Randall, Boston, Mass.

LOCKHEED AIRCRAFT CORP. 389
Hal Stebbins Inc., Los Angeles, Calif.

MARMAN PRODUCTS CO., INC. 393
West-Marquis, Inc., Los Angeles, Calif.

MAROTTA VALVE CO. 344
Paul M. Healy, Montclair, N. J.

MARQUARDT AIRCRAFT CO. 396
Heintz & Co., Inc, Los Angeles, Calif.

MARTIN, THE GLENN L., CO. 400
Vansant, Dugdale & Co., Baltimore, Md.

MINIATURE PRECISION BEARINGS INC. 399
Packard & Kraft, Worcester, Mass.

N. Y. AIR BRAKE CO. 397
Humbert & Jones, New York, N. Y.

NUTT-SHEL CO. 376
Welsh Hollander Advert., Los Angeles, Calif.

PANORAMIC RADIO PRODUCTS, INC. 343
Harold Marshall Advert. Co., New York, N. Y.

PHOTOCON RESEARCH PRODUCTS 377
George Burtt, Hollywood, Calif.

POTTER AERONAUTICAL CO. 398
Richard & Gunther, New York, N. Y.

REACTION MOTORS, INC. ..... inside front cover
399
Wm. von Zehle & Co., New York, N. Y.

REYNOLDS ELECTRICAL & ENGINEERING CO. 399

ROBINSON AVIATION INC. 391
Platt, Zachary & Sutton, Inc., New York, N. Y.

ROSÁN INC. 381
James Galloway, Los Angeles, Calif.

STATHAM LABORATORIES 388
Western Advert. Agency, Los Angeles, Calif.

SOUTHWEST PRODUCTS CO. 402
O. K. Fagan Advert. Agency, Los Angeles, Calif.

SUMMERS GYROSCOPE CO. 406
Byron H. Brown and Staff, Los Angeles, Calif.

TITEFLEX, INC. 347
John Falkner Arndt, Philadelphia, Pa.

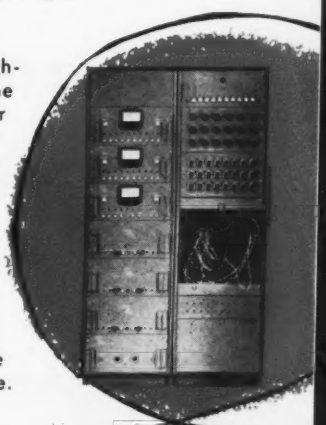
WATERTOWN DIV., N. Y. AIR BRAKE CO. 397
Humbert & Jones, New York, N. Y.

Save Engineering Time

ON • Equation Solving • System Simulation • Testing

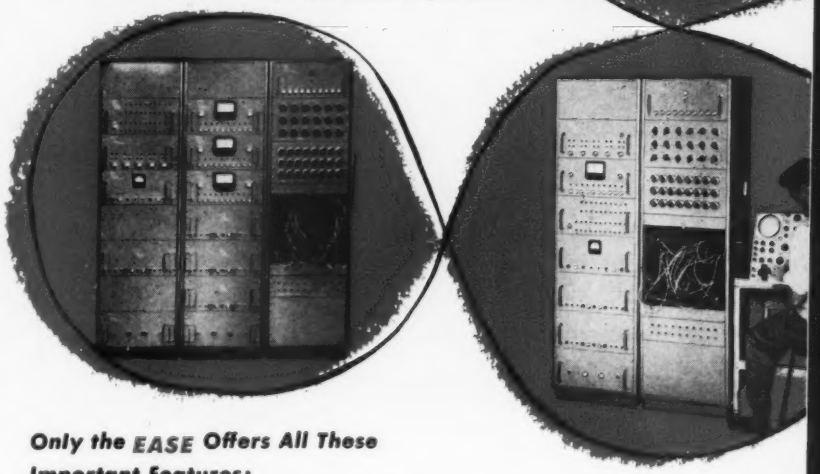
do it with.. **EASE***

***Electronic Analog Simulating Equipment**



As a computer, the EASE solves high-order differential equations in a fraction of the time required for slide-rule and calculator methods. As a system simulator, it can represent the functioning of an entire dynamic system. Effects of changing loads; forces and other variables can be determined rapidly and accurately.

In testing, a physical component may be studied in a complete system through the use of voltage transducers. In this manner, where the EASE simulates the associated equipment, fast yet thorough testing over a wide range of conditions is possible.



Only the EASE Offers All These Important Features:

- **SIMPLICITY** . . . any engineer or mathematician who can write the equation can solve it on the EASE with only a few hours' training.
- **VERSATILITY** . . . Additional components may be added at any time to extend its range of application. Its function generator can also serve as a resolver with an accuracy of better than 1%. It can be operated in real time or as a repetitive computer, or the problem can be "held" at any stage.
- **HIGH ACCURACY** . . . phase shift is less than 1° at 3500 cps. With a 10-channel amplifier stabilizer (optional), drift is less than 100 microvolts referred to grid. Meters on amplifiers permit extremely accurate potentiometer calibration.
- **COMPACTNESS** . . . unit requires less than 8 sq. ft. of floor space.
- **EASY INSTALLATION** . . . contains its own regulated power supply; normally merely plugs in to 20 amp, 110 v. 60-cycle a.c. line!
- **LOW COST** . . . the EASE is the world's first high-quality computer to be mass-produced in practical commercial form. The result is low cost without sacrifice in utility or quality.

For complete data, request Bulletin F11

Berkeley *division*
BECKMAN INSTRUMENTS INC.
2200 WRIGHT AVE., RICHMOND, CALIF.

M-20

NEW DISCOVERY IN INSTRUMENTATION

(reading time · 15 seconds)

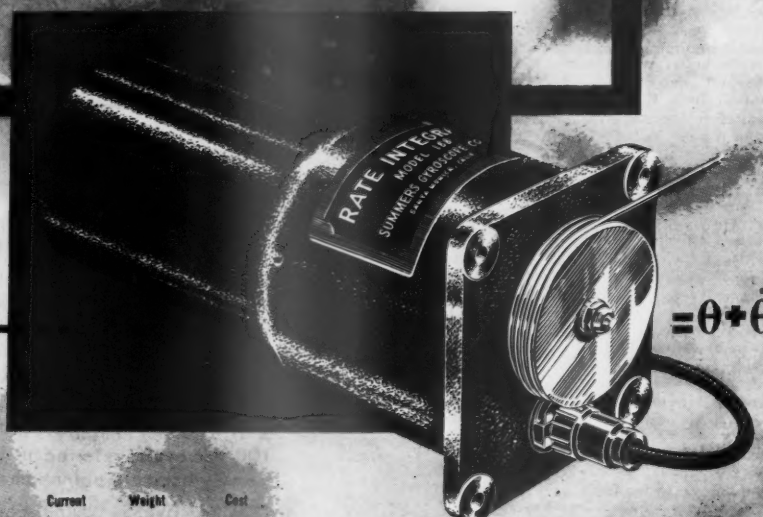
Yesterday

Directional Gyro
Torquer
Cager
Pickoff

+ Rate Gyro
Pickoff
Damper

+ Mixing Circuitry

$= \theta + \dot{\theta}$



TODAY..

$= \theta + \dot{\theta}$

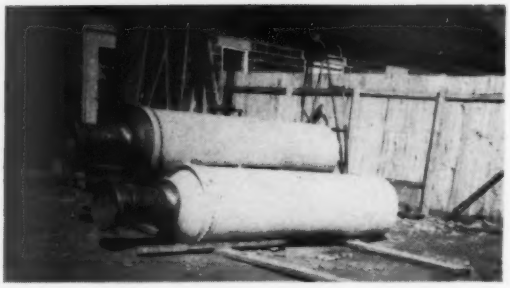
	Reliability	Drift	Current	Weight	Cost
yesterday	.950	.5 /Min	3 amps	12 lbs	
today	.999	.1 /Min	.3 amps	3 lbs.	

SUMMERS
GYROSCOPE
COMPANY
2328 BROADWAY · SANTA MONICA, CALIFORNIA

EXCELCO DEVELOPMENTS

INCORPORATED

MILL STREET - BOX 230 - SILVER CREEK, N. Y.

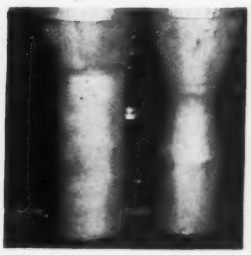


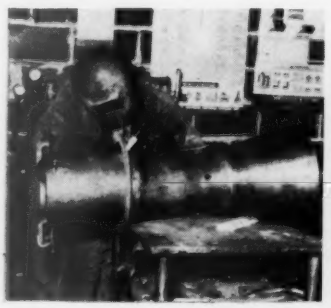
For Skill And Precision
In The Development
And Manufacture Of ...

ROCKET MOTORS
and
GUIDED MISSILE COMPONENTS

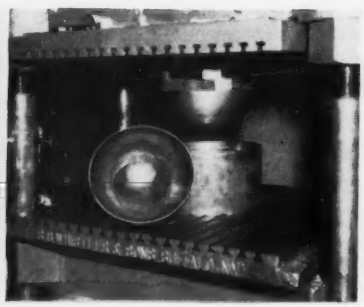
Fabricators Of

- COMPLETE ROCKET MOTORS
- SPHERES FOR PRESSURE TANKS
- NOZZLES OF ALL TYPES
- INNER & OUTER THROAT SECTIONS
- AIRCRAFT SEATS & BULKHEADS
- ELECTRONIC CHASSIS & DETAIL ASSY
- LOX BOILERS
- TORUS TANKS
- SPECIAL MACHINING

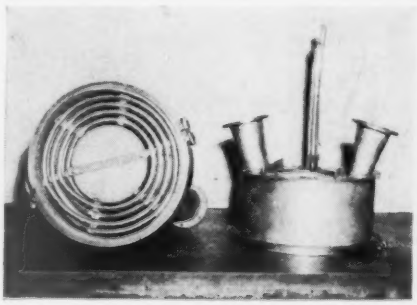


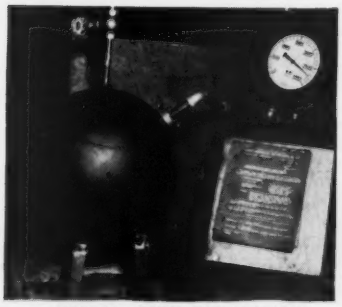


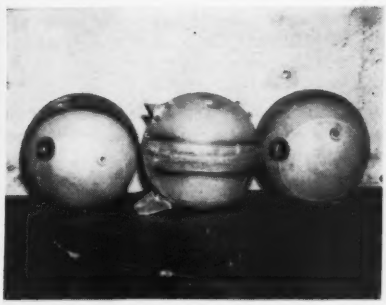
HELI-ARC WELDING



FORMING OF STOCK TO PRECISION DIMENSIONS







TESTING AFTER FABRICATION

EXCELCO DEVELOPMENTS INC.

SUPERIOR PERFORMANCE

On 3 Cycles to 20 Megacycles

The
LA-239 C
Oscilloscope

DATA

1. Wider Bandwidth: Complex waves from 5 Cycles to 15 Megacycles. Sine waves from 3 Cycles to 20 Megacycles.

2. Extended Sweep Frequencies: Linear from 10 Cycles to 20 Megacycles internally synchronized. Triggered sweep, from single random impulses to irregular pulse-intervals up to as high as 6 Megacycles.

3. Square Wave Response: Rise time 0.042 Microseconds; only 5% droop on flat-topped pulses as long as 30,000 Microseconds duration.

4. Greater Stability: Electronically regulated power supplies throughout to maintain accuracy and constant operation under varying line conditions or line surges. You can display surges on the line from which Model LA-239C is being powered without distortion of the trace!

5. Higher Signal Sensitivity: Maximum sensitivity without Probe: 10.4 millivolts. With Probe: 100 millivolts. (Maximum signals, 125 V. Peak and 450 V. Peak respectively.)

6. Timing Markers: Interval Markers of 0.2; 1; 5; 20; 100; 500; or 2,000 Microseconds may be superimposed on the trace for the accurate measurement of the time base.

7. Voltage Calibration: Signal amplitude is compared against a 1,000 cycle square wave (generated internally) the amplitude of which is controlled by a step-and-slide attenuator calibrated in peak volts. (A jack is provided to deliver 40V Peak for use in calibrating other instruments.)

8. Sweep Delay: Any portion of the sweep longer than a 10 Microsecond section may be expanded by 10:1 for detailed study of that portion of the signal.

9. Power Source: 110 to 130 V AC; from 50 to 1,000 cycles. 295 Watts. (Fused at 4 Amperes.)

10. Dimensions: In Bench Cabinet: 19½ in. Wide; 15¼ in. High; 16¾ in. Deep. In Rack Mounting (With cabinet removed to fit standard relay rack): 19½ in. Wide; 14 in. High.

THE LAVOIE MODEL LA-239C has been designed to surpass the high performance of the TS-239A/UP, which has been the standard test oscilloscope for the Armed Services since its introduction. Model LA-239C is the result of a long period of research and development which has included the study of new tubes, new circuits, and new techniques. Rugged design has been combined with functional simplicity to produce an instrument as attractive as it is efficient.

To create a circuit that will produce a certain complex wave form, or study transients and pulse phenomena, no better precision instrument is available today.

Lavoie Laboratories take pride in offering this precision oscilloscope as the combination of engineering perfection and manufacturing skill.

UHF

Lavoie Laboratories, Inc.
MORGANVILLE, NEW JERSEY
DESIGNERS AND MANUFACTURERS OF ELECTRONIC EQUIPMENT

3888 . 2